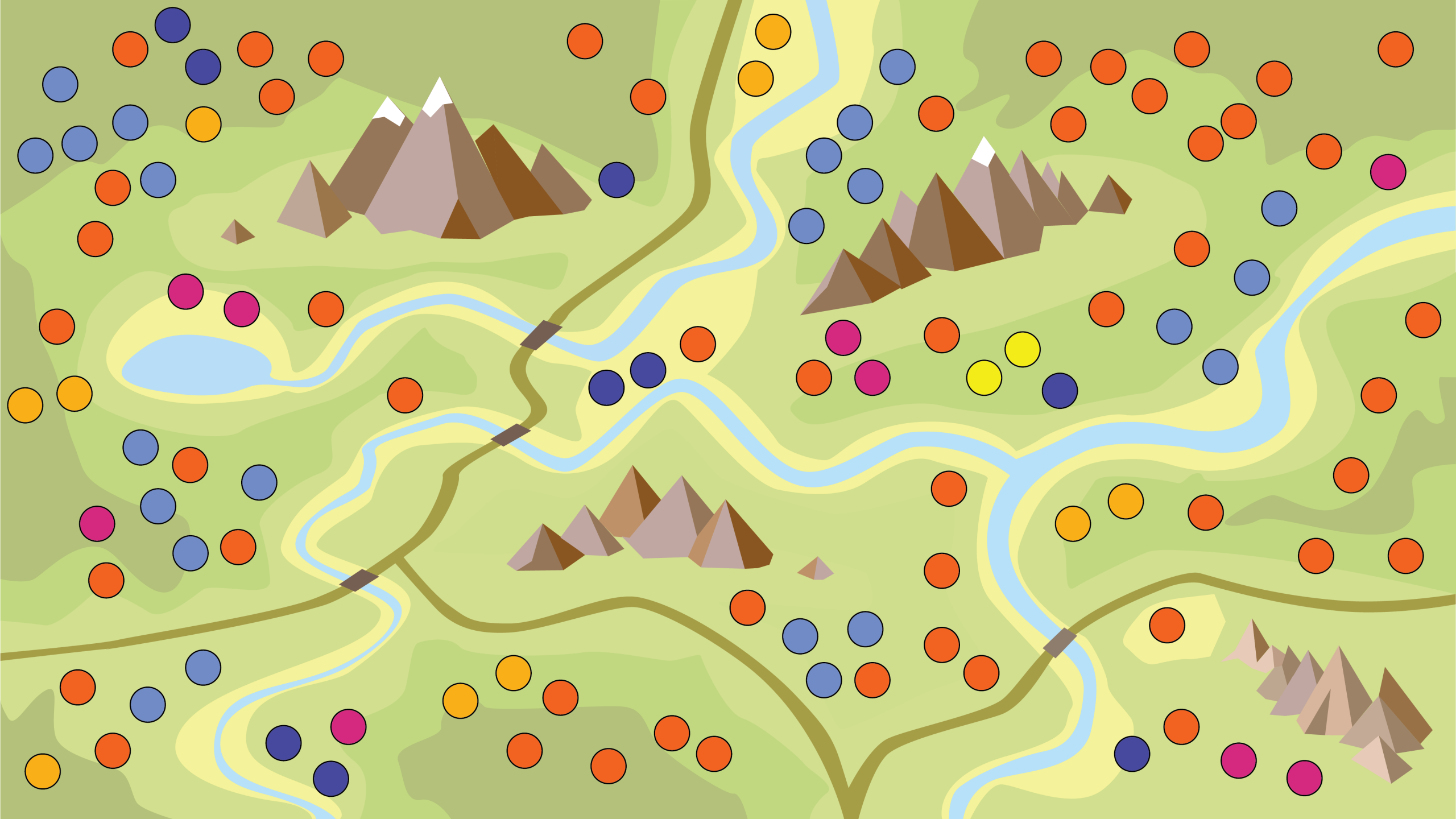
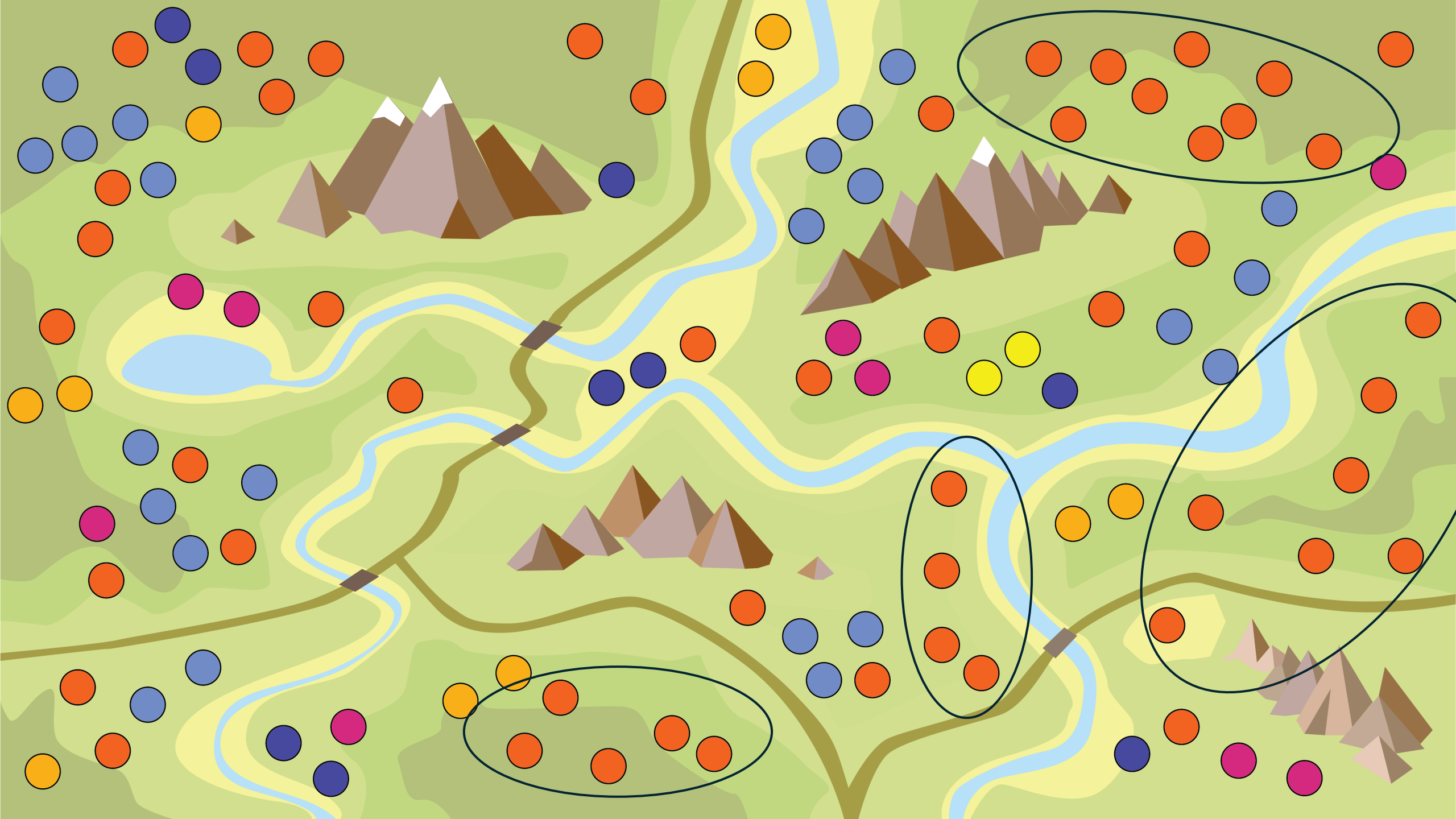


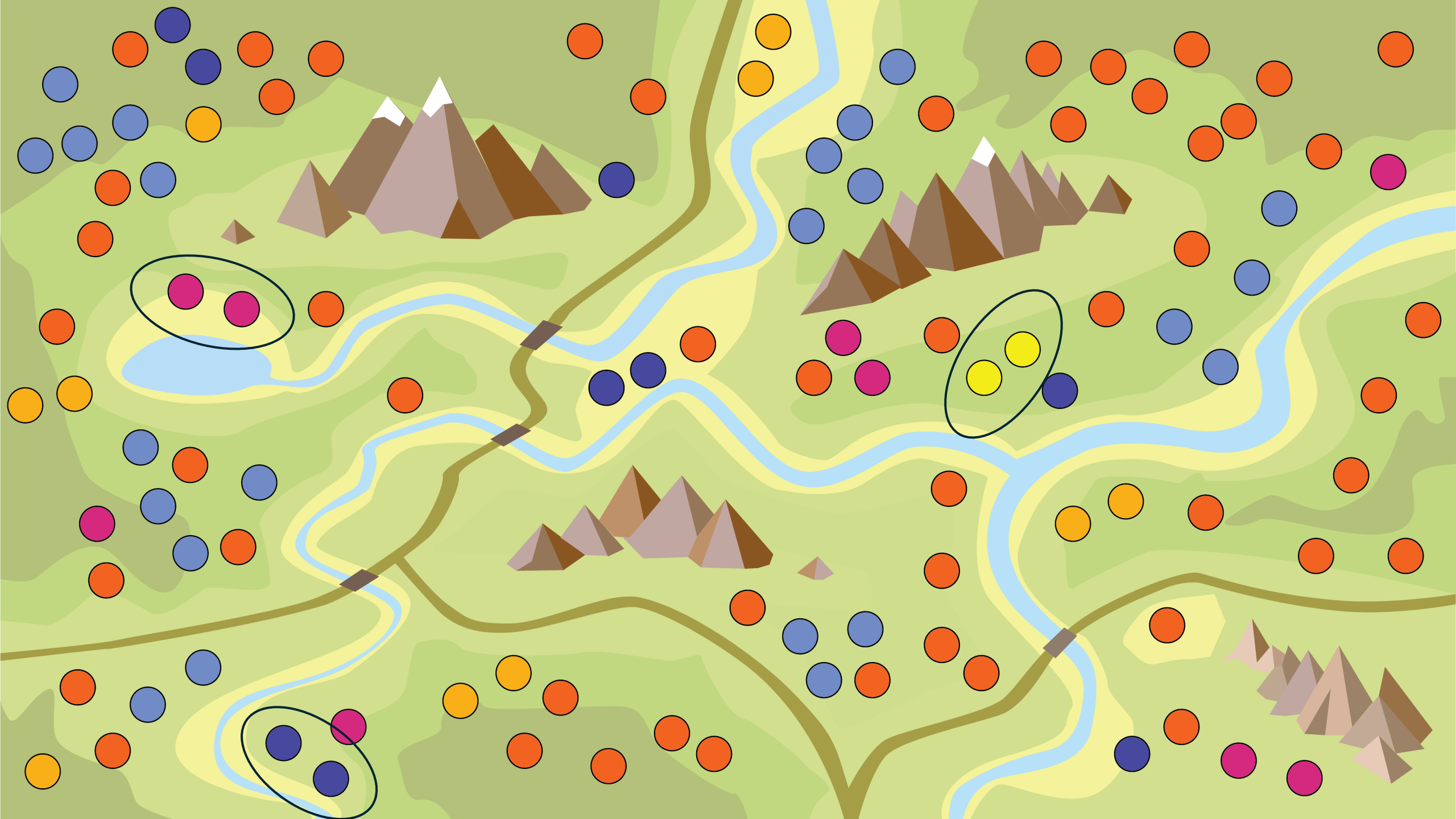
A Global Model of Discretized Rarity



Alivia G. Nytko, Brian C. O'Meara,
Joe K. Bailey









Geographic Range

Large

Small

Habitat Specificity

Wide

Narrow

Wide

Narrow

**Local Population
Size**

Large

Small

Geographic Range

Large

Small

Habitat Specificity

Wide

Narrow

Wide

Narrow

**Local Population
Size**

Large

Small

			Endemic

Geographic Range

Large

Small

Habitat Specificity

Wide

Narrow

Wide

Narrow

**Local Population
Size**

Large

Small

Sparse			

Geographic Range

Large

Small

Habitat Specificity

Wide

Narrow

Wide

Narrow

Large

Local Population Size

Small

Functional Rarity: Trait Distinctiveness

Phylogenetic Rarity: Evolutionary Distinctiveness

Geographic Range		Large		Small	
Habitat Specificity		Wide	Narrow	Wide	Narrow
Local Population Size	Large				
	Small				

Functional Rarity: Trait Distinctiveness

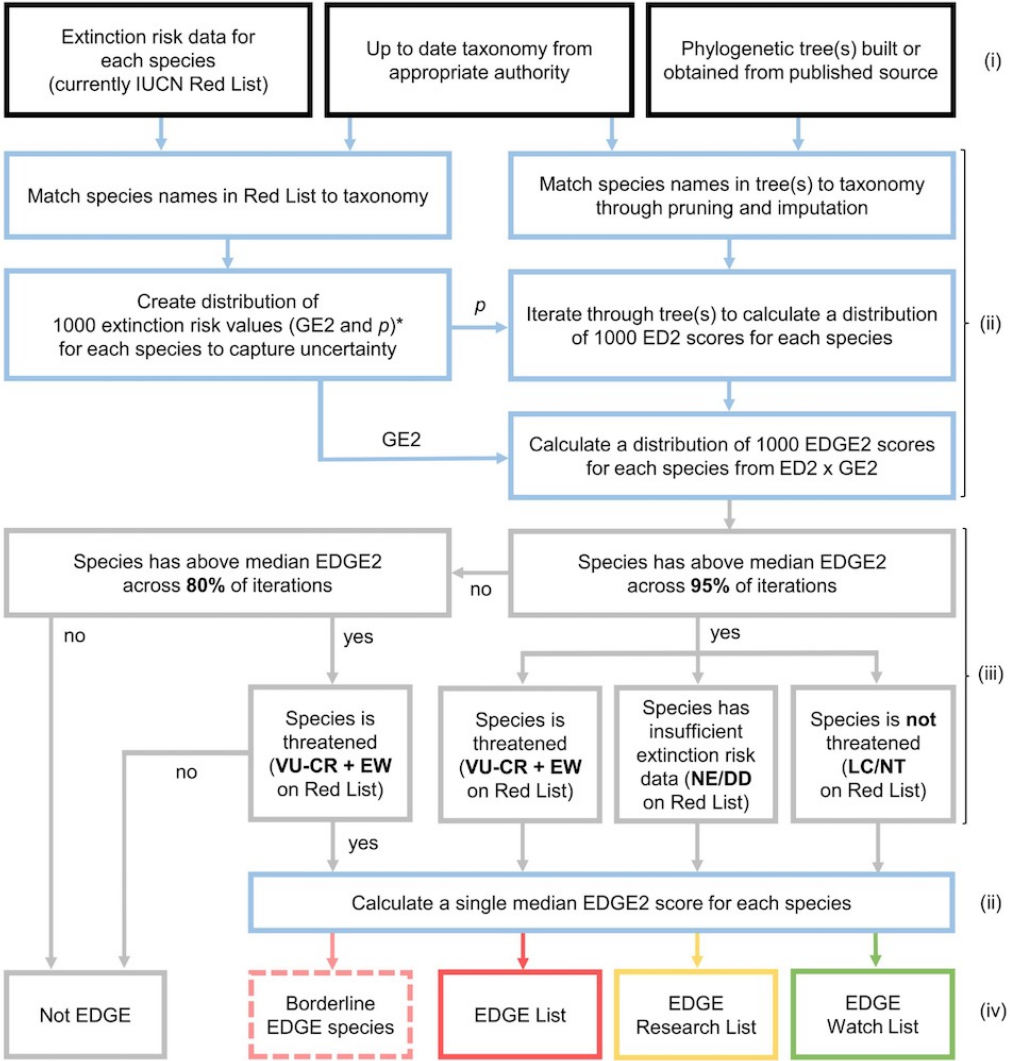
Phylogenetic Rarity: Evolutionary Distinctiveness

Geographic Range		Large		Small	
Habitat Specificity		Wide	Narrow	Wide	Narrow
Local Population Size	Large				
	Small				

Functional Rarity Framework

		<u>Species Frequency</u>				
		Restricted		Widespread		
<u>Species Traits</u>	Geographic Restrictedness	Abundant	Scarce	Abundant	Scarce	
	Unique	Distinct	R			
		Indistinct				
	Shared	Distinct				
		Indistinct				C
Geographic Uniqueness		Geographic Distinctiveness				

EDGE Protocol



Functional Rarity Framework

EDGE Protocol

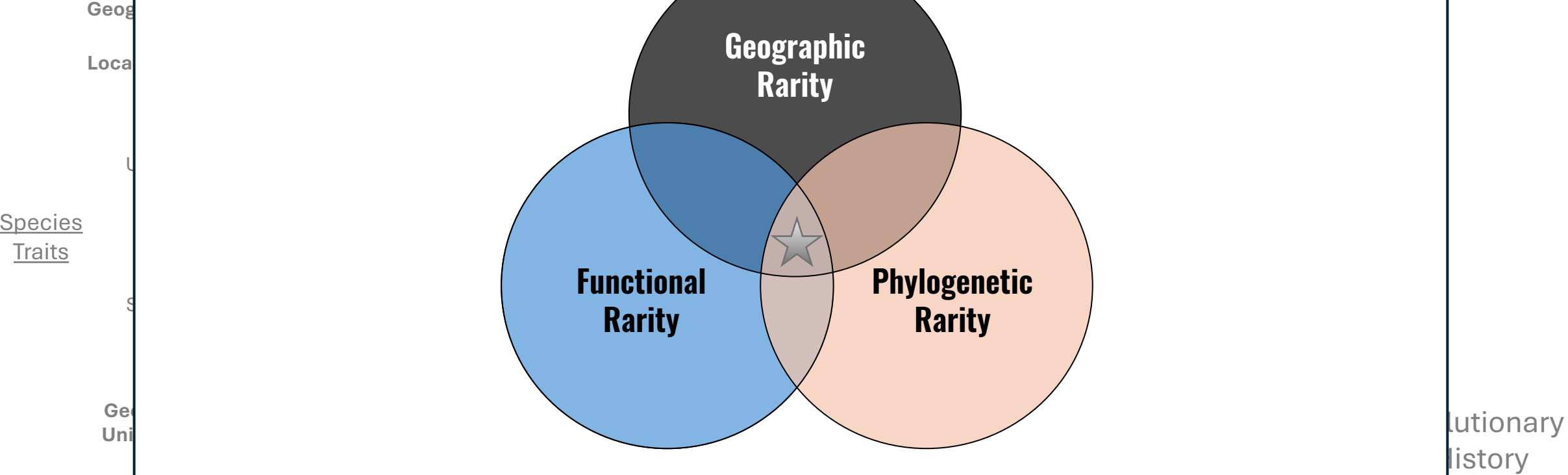


Table 1

Summary of the links between rarity, distinctiveness, originality, and diversity developed in the main text with diversity indices Q , ${}^{\alpha}K$ and ${}^{\alpha}K^*$.

	Quadratic diversity ^a	First parametric extension ^a	Second parametric extension ^a
Species-level			
FP-distinctiveness	$D_j = \sum_{i \neq j}^N \frac{p_i}{1 - p_j} d_{ij}$	${}^{\alpha}D_j = \frac{1 - \left(\sum_{i=1}^N p_i s_{ij}\right)^{\alpha-1}}{1 - p_j^{\alpha-1}}$	${}^{\alpha}D_j^* = \frac{\sum_{c=1}^{N-1} u_{c j} \left(1 - p_{c j}^{\alpha-1}\right)}{\left(1 - p_j^{\alpha-1}\right)}$
Abundance-based rarity	$\rho_j = (1 - p_j)$	${}^{\alpha}\rho_j = \frac{1 - p_j^{\alpha-1}}{\alpha - 1}$	${}^{\alpha}\rho_j = \frac{1 - p_j^{\alpha-1}}{\alpha - 1}$
Effective originality	$O_j = U_j \times \rho_j$	${}^{\alpha}O_j = {}^{\alpha}D_j \times {}^{\alpha}\rho_j$	${}^{\alpha}O_j^* = {}^{\alpha}D_j^* \times {}^{\alpha}\rho_j$
Community-level			
Species diversity ^b	$S = \sum_{j=1}^N p_j \rho_j$ $= 1 - \sum_{i=1}^N p_i^2$	${}^{\alpha}S = \sum_{j=1}^N p_j \left({}^{\alpha}\rho_j\right)$ $= \frac{1 - \sum_{j=1}^N p_j^{\alpha}}{\alpha - 1}$	${}^{\alpha}S = \sum_{j=1}^N p_j \left({}^{\alpha}\rho_j\right)$ $= \frac{1 - \sum_{j=1}^N p_j^{\alpha}}{\alpha - 1}$
FP-diversity	$Q = \sum_{j=1}^N p_j O_j$ $= \sum_{i=1}^N \sum_{j=1}^N p_i p_j d_{ij}$	${}^{\alpha}K = \sum_{j=1}^N p_j \left({}^{\alpha}O_j\right)$ $= \sum_{j=1}^N p_j \times \frac{1 - \left(\sum_{i=1}^N p_i s_{ij}\right)^{\alpha-1}}{\alpha - 1}$	${}^{\alpha}K^* = \sum_{j=1}^N p_j \left({}^{\alpha}O_j^*\right)$ $= \sum_{j=1}^N p_j \times \frac{\sum_{c=1}^{N-1} u_{c j} \left(1 - p_{c j}^{\alpha-1}\right)}{\alpha - 1}$
Species equivalents ^c	$E = 1 / \sum_{i=1}^N \sum_{j=1}^N p_i p_j s_{ij}$	${}^{\alpha}E = \left(\sum_{j=1}^N p_j \left(\sum_{i=1}^N p_i s_{ij}\right)^{\alpha-1}\right)^{\frac{1}{1-\alpha}}$	${}^{\alpha}E^* = \left(\sum_{j=1}^N p_j \left(1 - \frac{\max_i(d_{ij})}{d_{\max}} + \sum_{c=1}^{N-1} \frac{u_{c j}}{d_{\max}} p_{c j}^{\alpha-1}\right)\right)^{\frac{1}{1-\alpha}}$



2.18

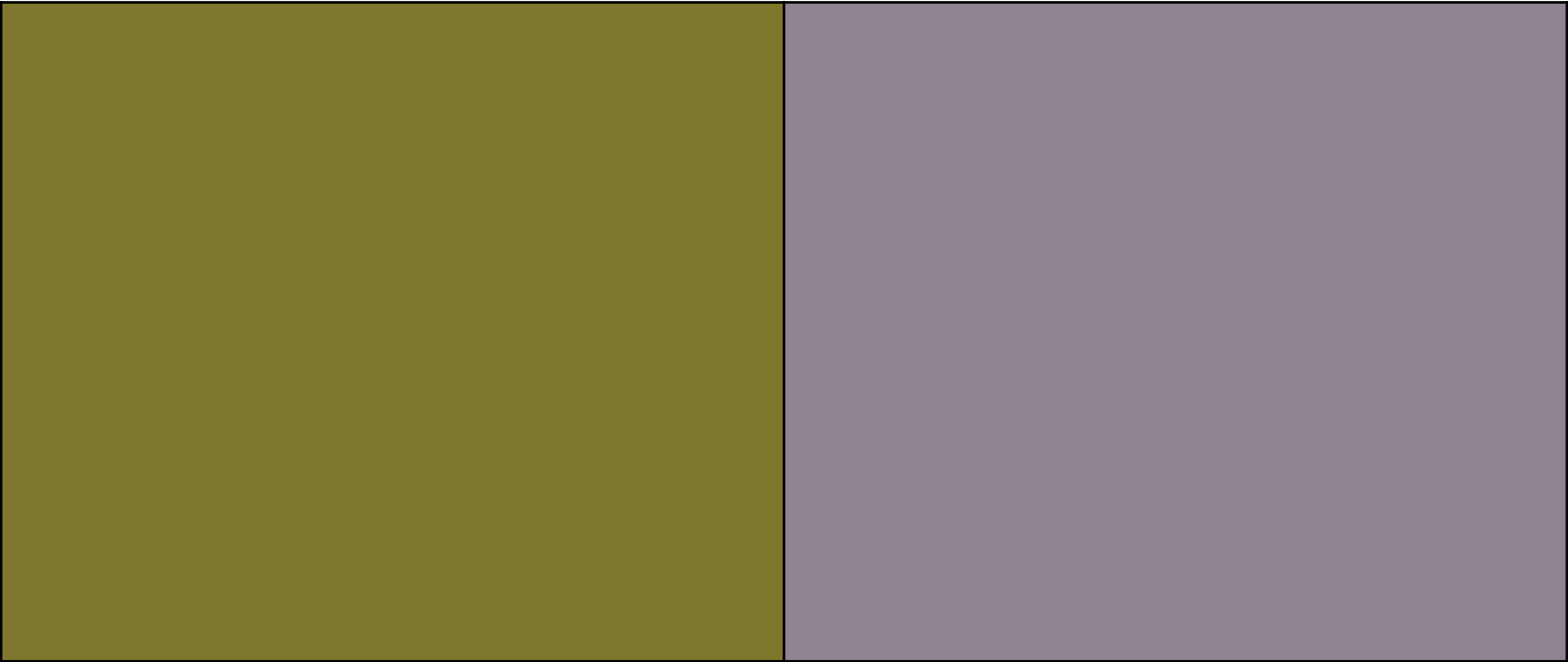


2.38

Range Size

Large

Small



Range Size

Large

Small

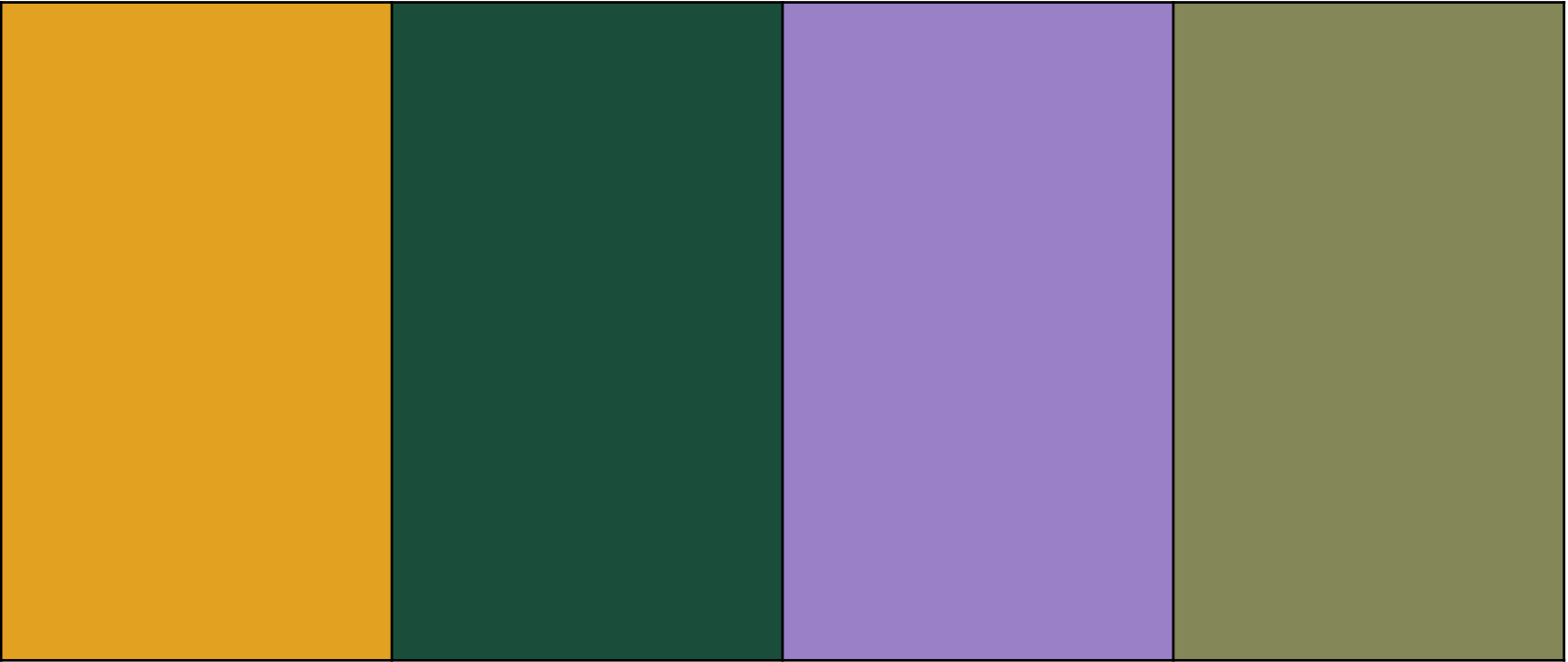
**Functional
Distinctiveness**

Distinct

Indistinct

Distinct

Indistinct



Range Size

Large

Small

**Functional
Distinctiveness**

Distinct

Indistinct

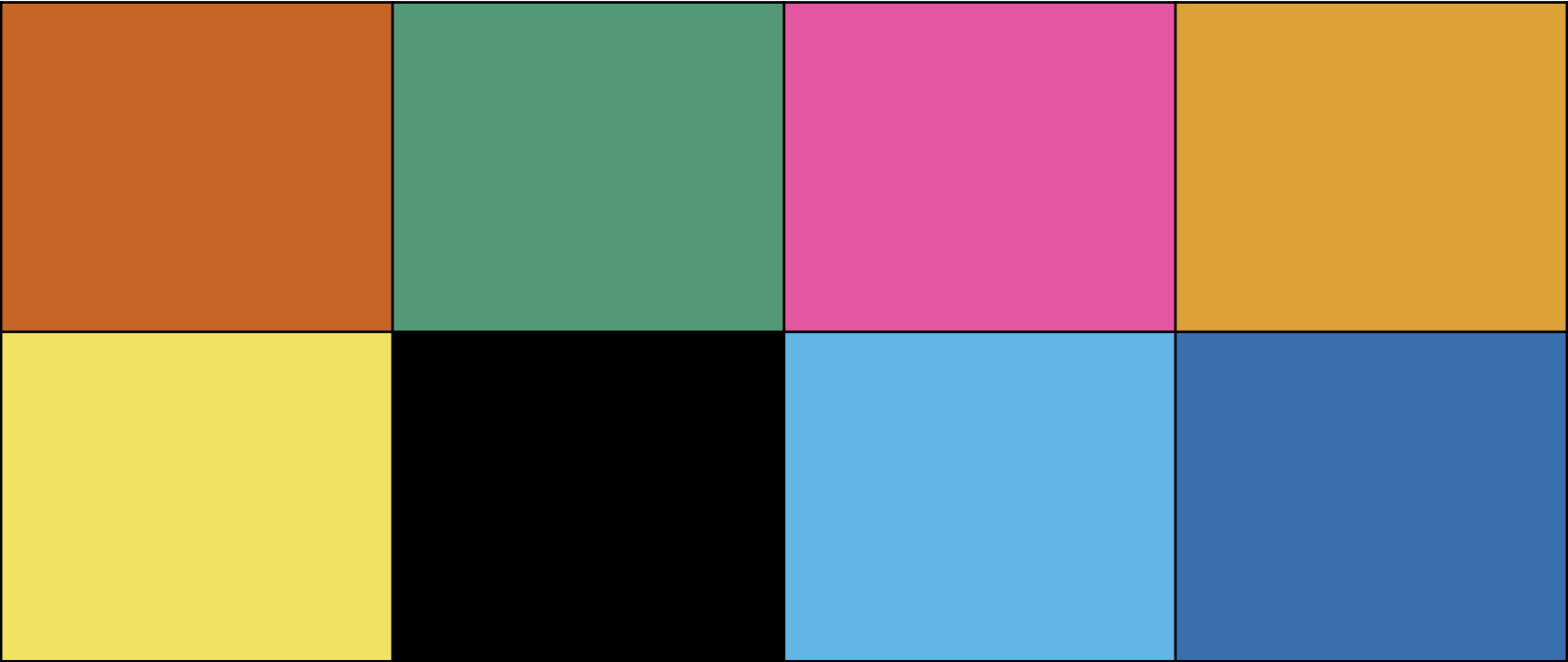
Distinct

Indistinct

**Evolutionary
Distinctiveness**

Distinct

Indistinct



Range Size

Large

Small

**Functional
Distinctiveness**

Distinct

Indistinct

Distinct

Indistinct

Distinct

**Evolutionary
Distinctiveness**

Indistinct

Indicator	Adaptable Survivor	Classically Rare	Relict
High Invasive Potential	Common	Endemic	Environmentally Rare



Environmentally Rare

- High stress tolerance
- General mutualisms
- Recent speciation

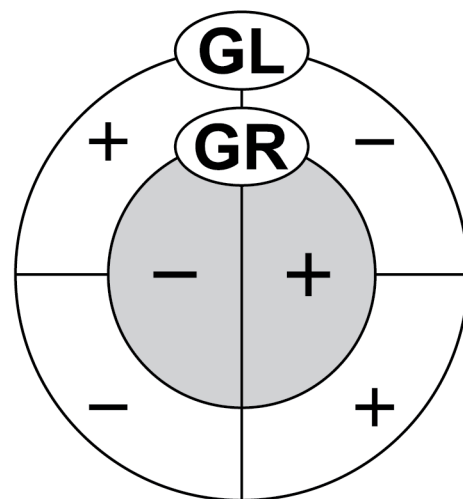


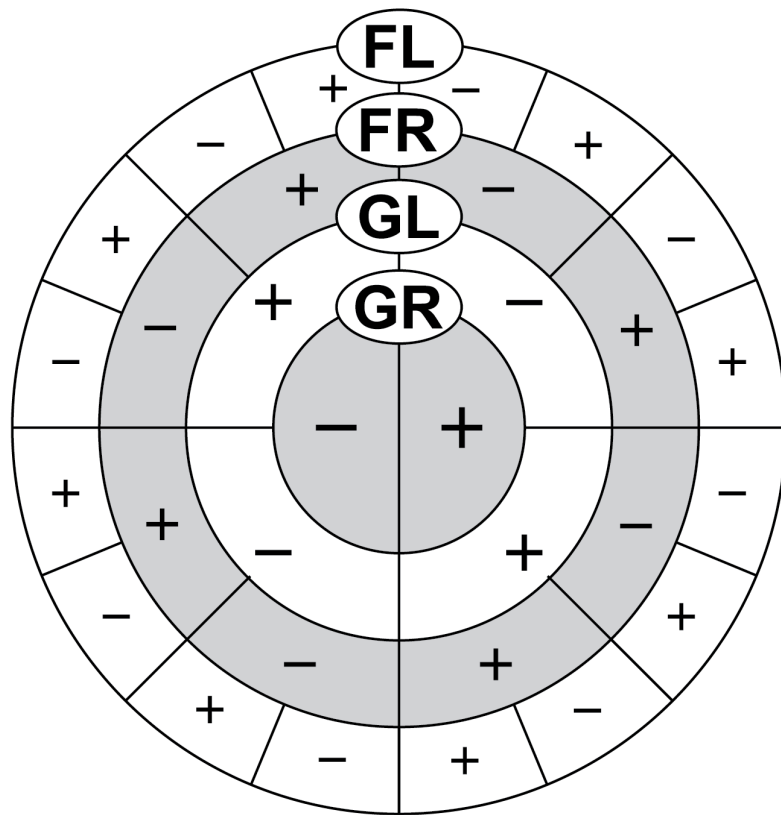
Indicator

- Indicative of ecosystem health
- Disruptive outside of native range

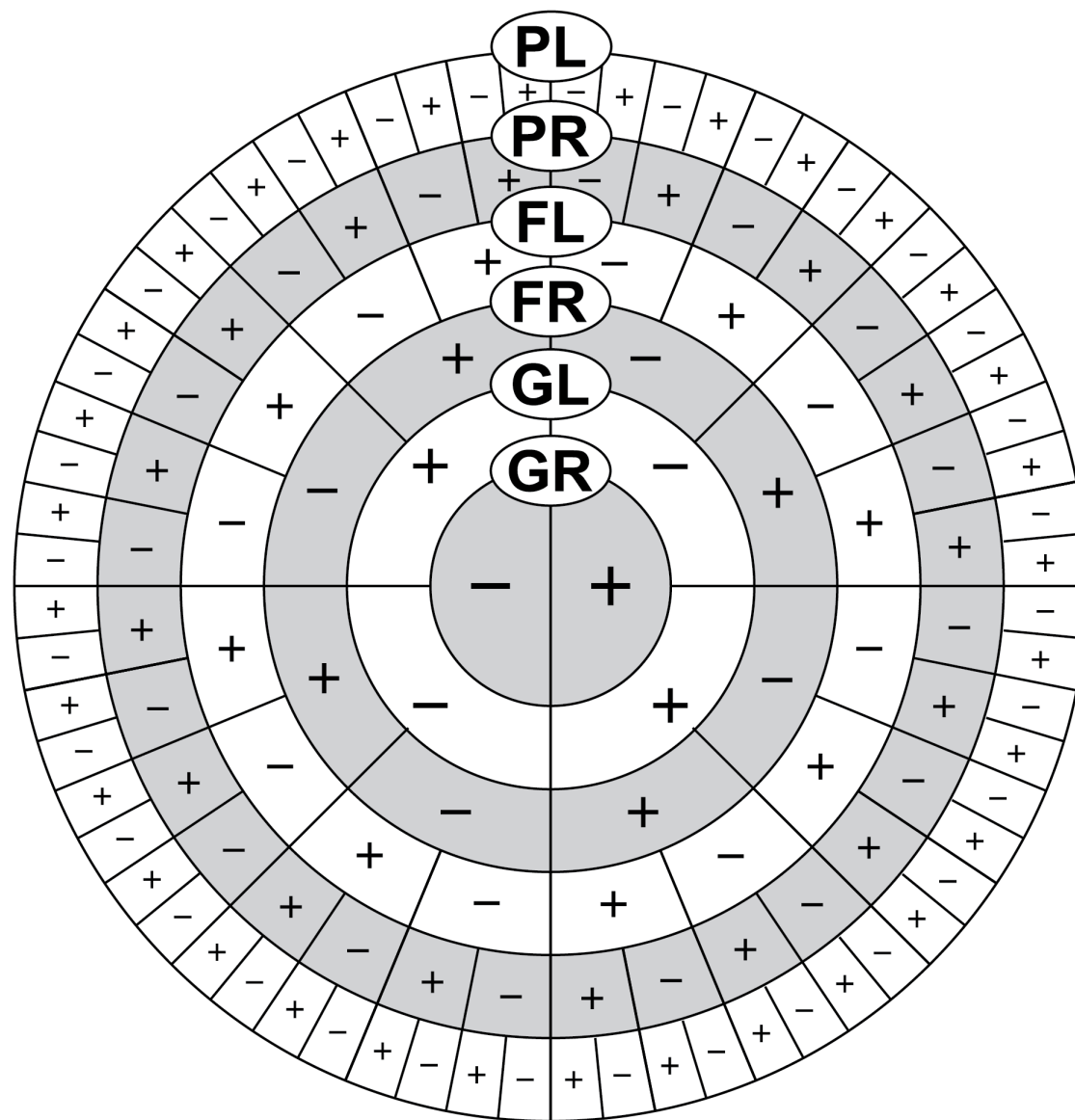
What is the best definition of discrete rarity?



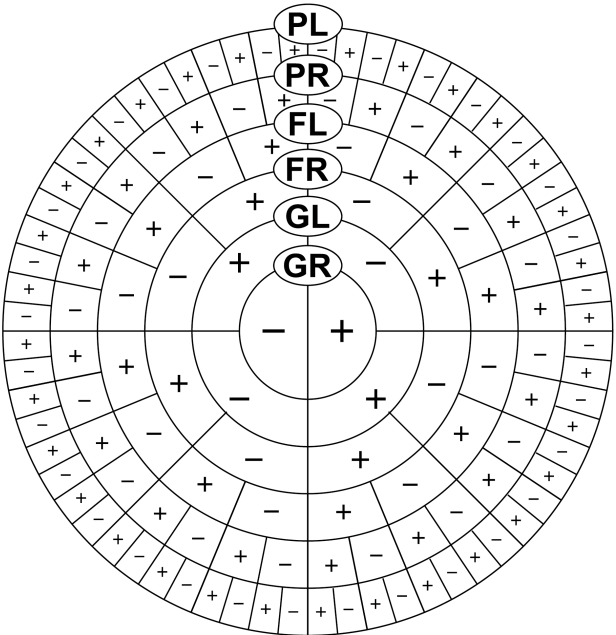




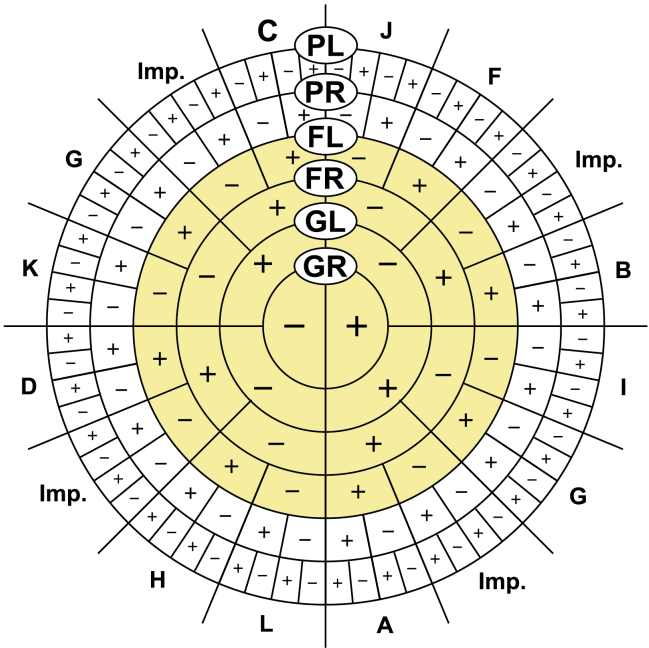
The Global Model of Discretized Rarity (GDR)



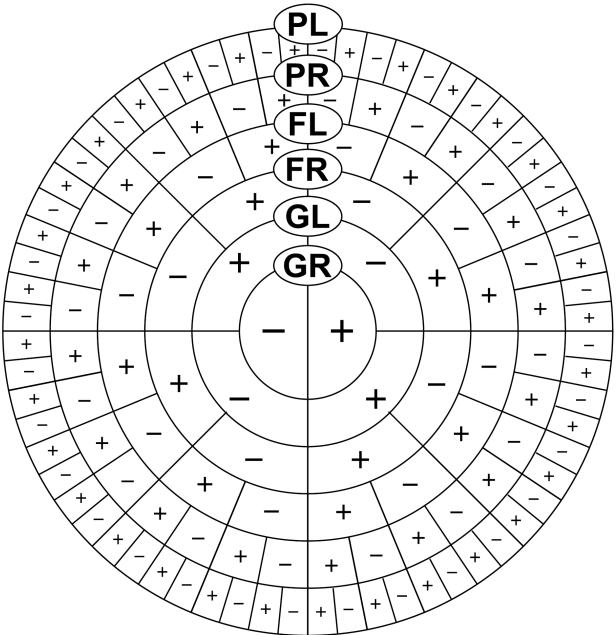
Global Model of Discretized Rarity



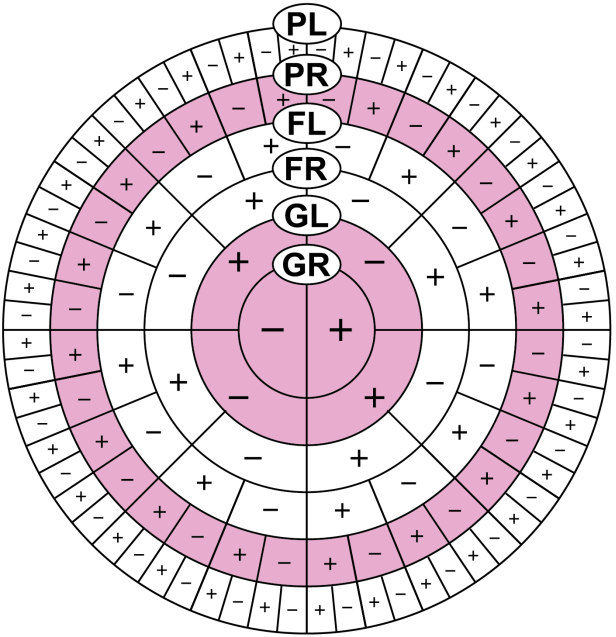
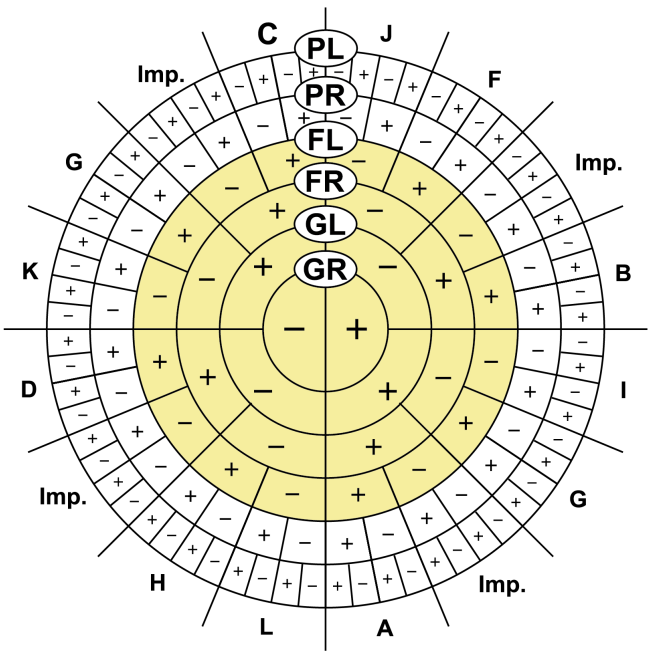
Functional Rarity Framework



Global Model of Discretized Rarity

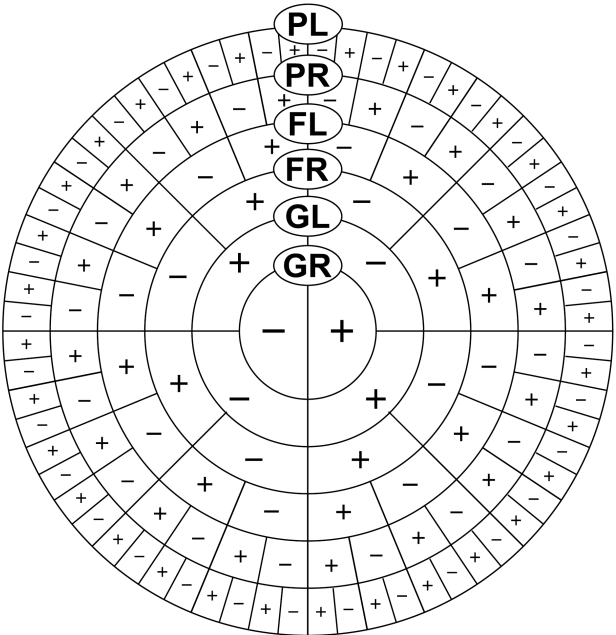


Functional Rarity Framework

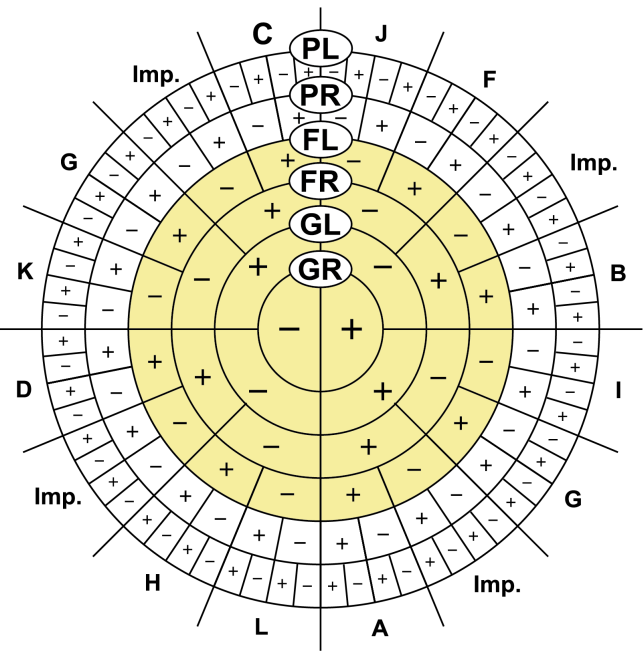


EDGE Protocol

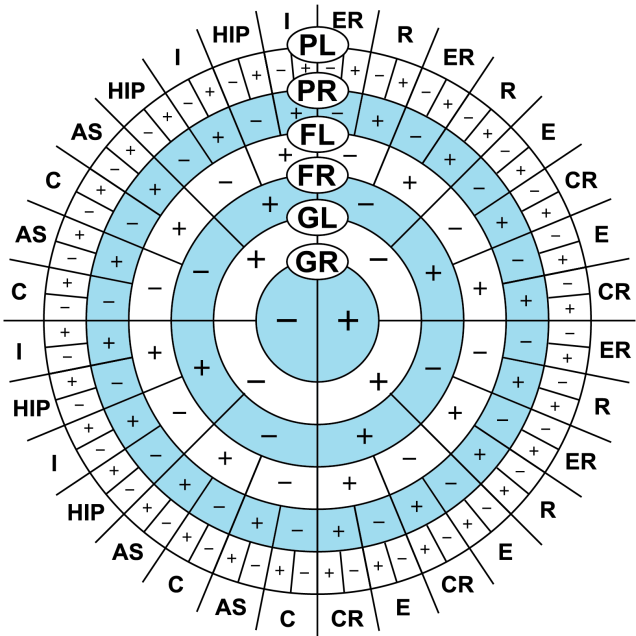
Global Model of Discretized Rarity



Functional Rarity Framework



Eco-Evolutionary Rarity



EDGE Protocol

Exploring the restrictions of GDR

1

Can we use rarity as a
tool to understand
ecological processes?
How?

2

Are rarity types robust
and ecologically
relevant?

Guide the selection and application of GDR restrictions



Data availability



Research/Management
Objectives



Methods & ability to explain
biological processes



Building a *working* global model



Building a *working* global model

Regional

GR = Extent of occurrence or taxonomic frequency

FR = Minimum or mean Euclidean distance

PR = Evolutionary Distinctiveness



Local

GL = Abundance

FL = Mean abundance weighted distance

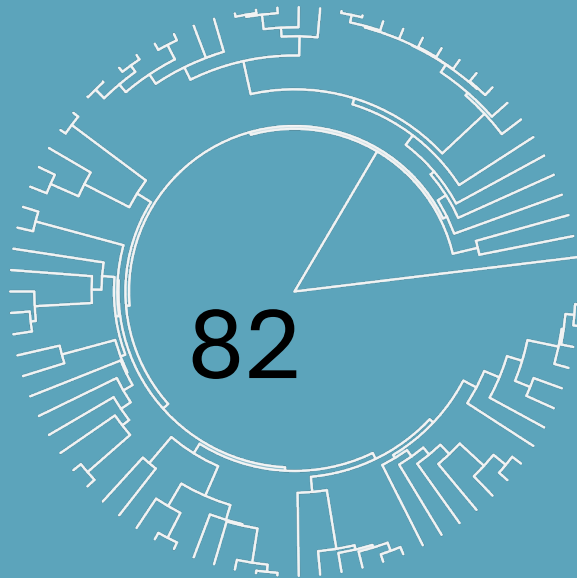
PL = Phylogenetic abundance weighted or unweighted mean pair-wise distance

British Plant Species

- Rabinowitz rarity was classified for 177 British flora in 1986.

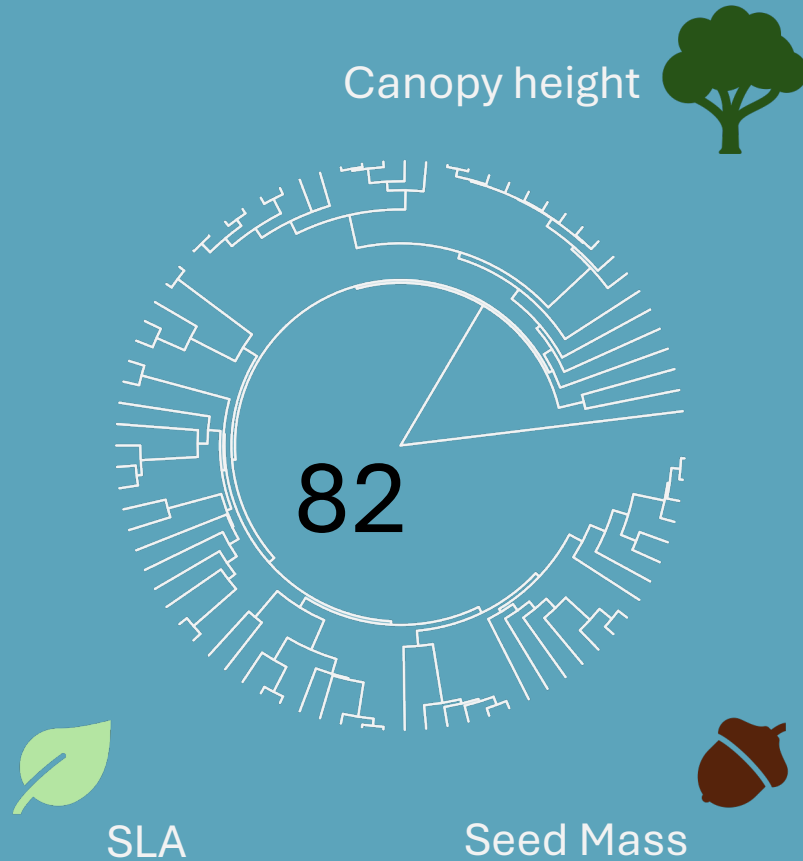
British Plant Species

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British Plant Species

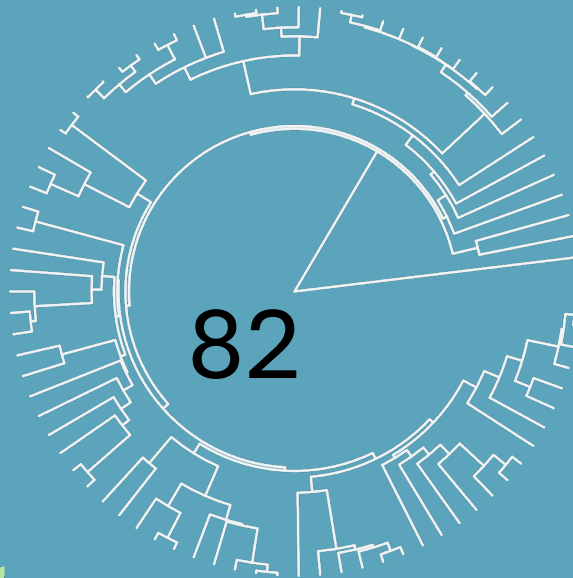
- Rabinowitz rarity was classified for 177 British flora in 1986.



British Plant Species

- Rabinowitz rarity was classified for 177 British flora in 1986.

Canopy height

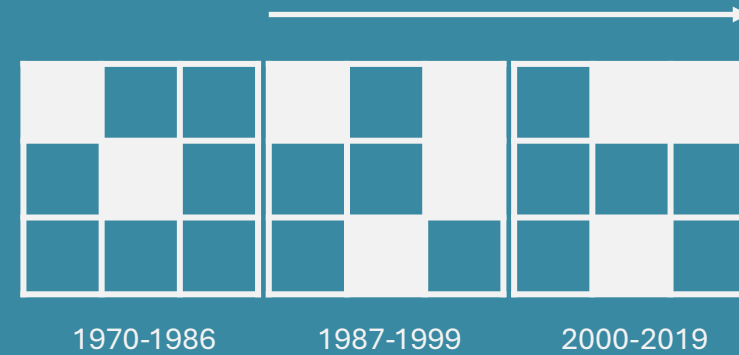


SLA



Seed Mass

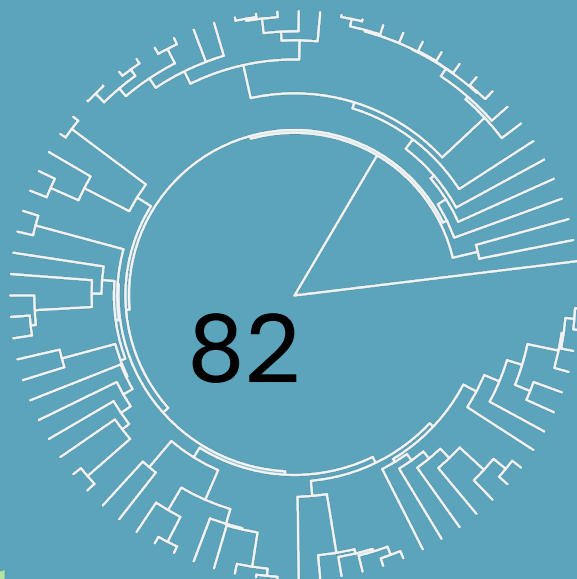
Plant Atlas 2020



British Plant Species

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Canopy height

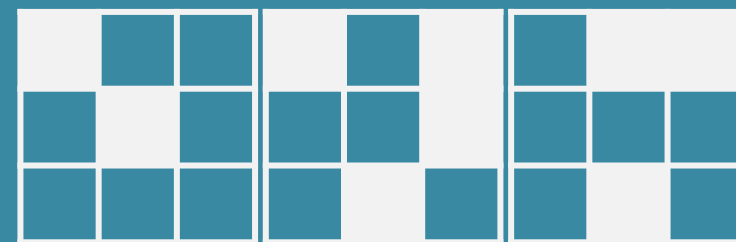


SLA



Seed Mass

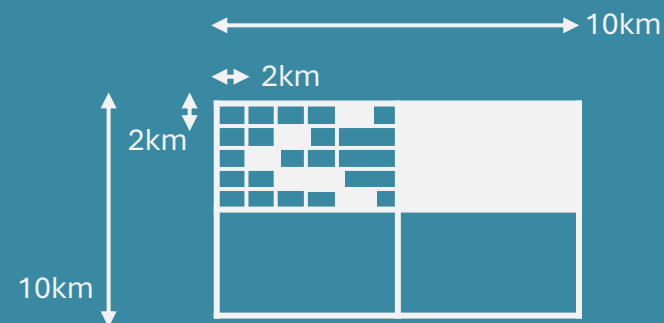
Plant Atlas 2020



1970-1986

1987-1999

2000-2019

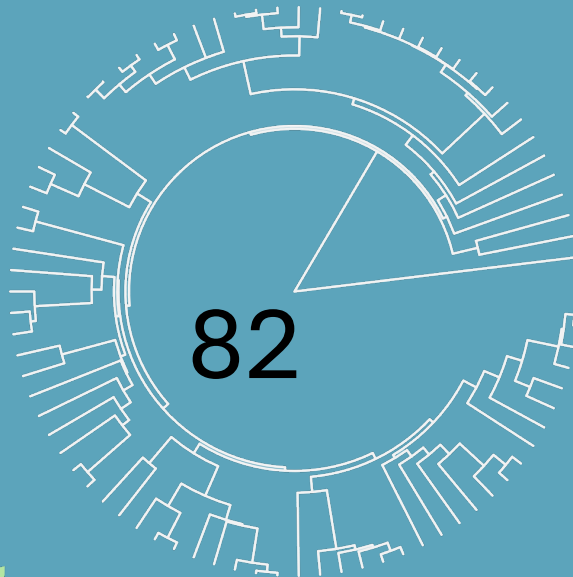


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Canopy height

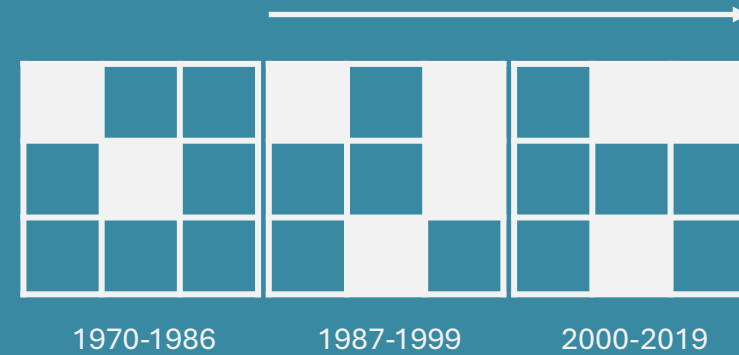


SLA



Seed Mass

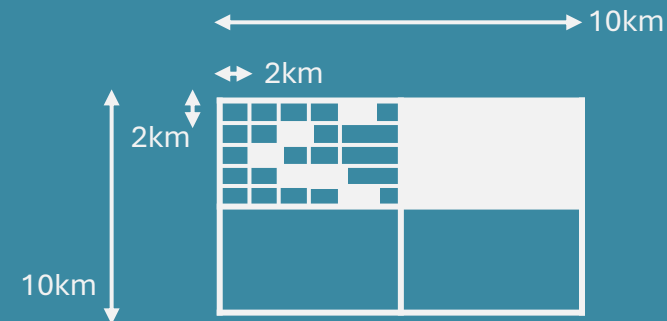
Plant Atlas 2020



1970-1986

1987-1999

2000-2019



1970-1986

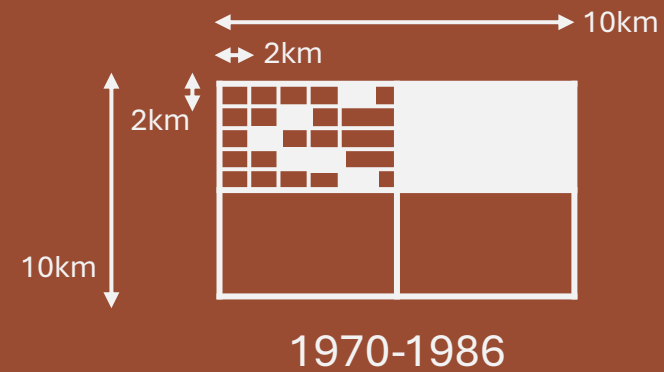
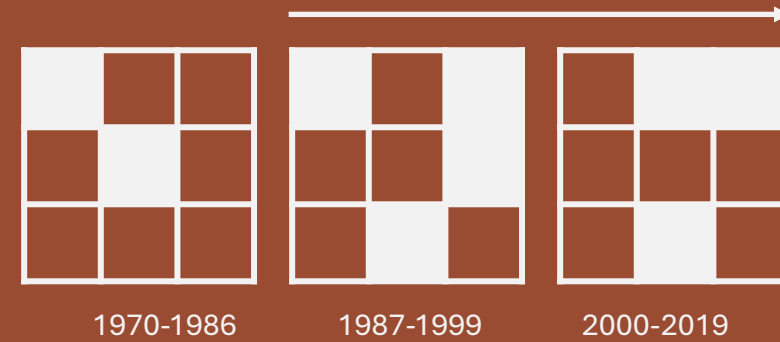


Flowering Window

British Plant Species



Plant Atlas 2020



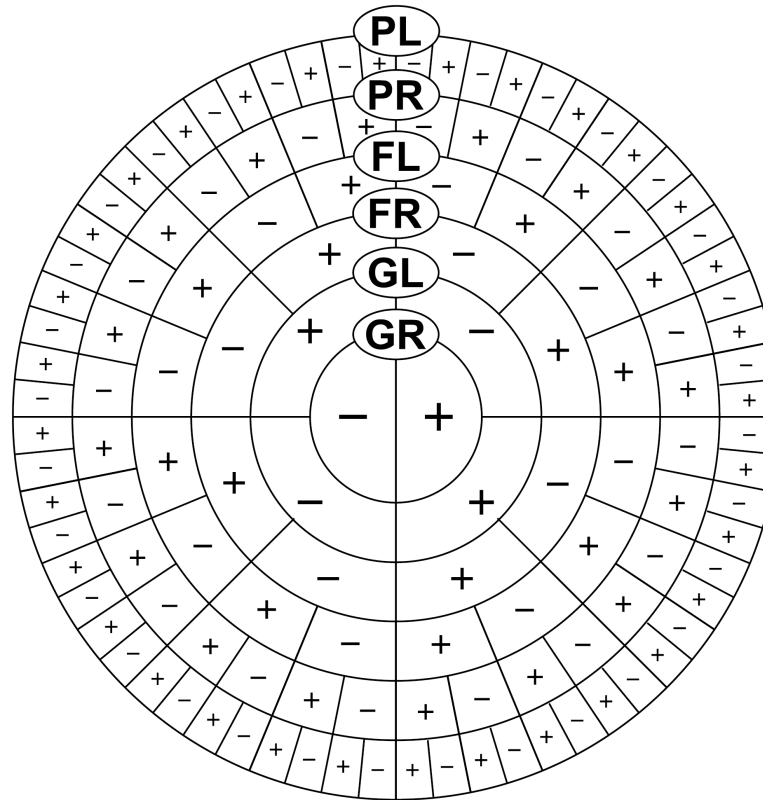
Flowering Window

82 Species

1011 Species

4 Thresholds X 2 GR X 2 FR X 2 PL = 32 Methods

63 Restrictions



82 Species

1011 Species

4 Thresholds X 2 GR X 2 FR X 2 PL = 32 Methods

63 Restrictions

For each method:

Distribution Change ~ Restriction

82 Species

1011 Species

4 Thresholds X 2 GR X 2 FR X 2 PL = 32 Methods

63 Restrictions

For each method:

Distribution Change ~ Restriction

Flowering Phenology ~ Restriction

82 Species

1011 Species

4 Thresholds X 2 GR X 2 FR X 2 PL = 32 Methods
63 Restrictions

For each method:

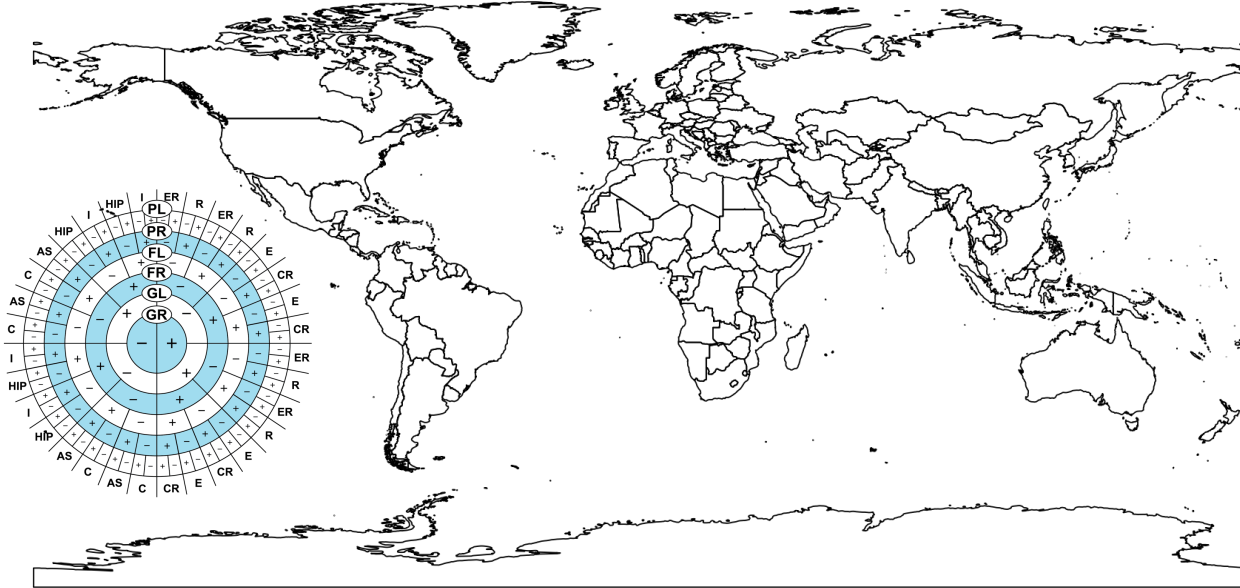
Distribution Change ~ Restriction

Flowering Phenology ~ Restriction

Best Method per Restriction

&

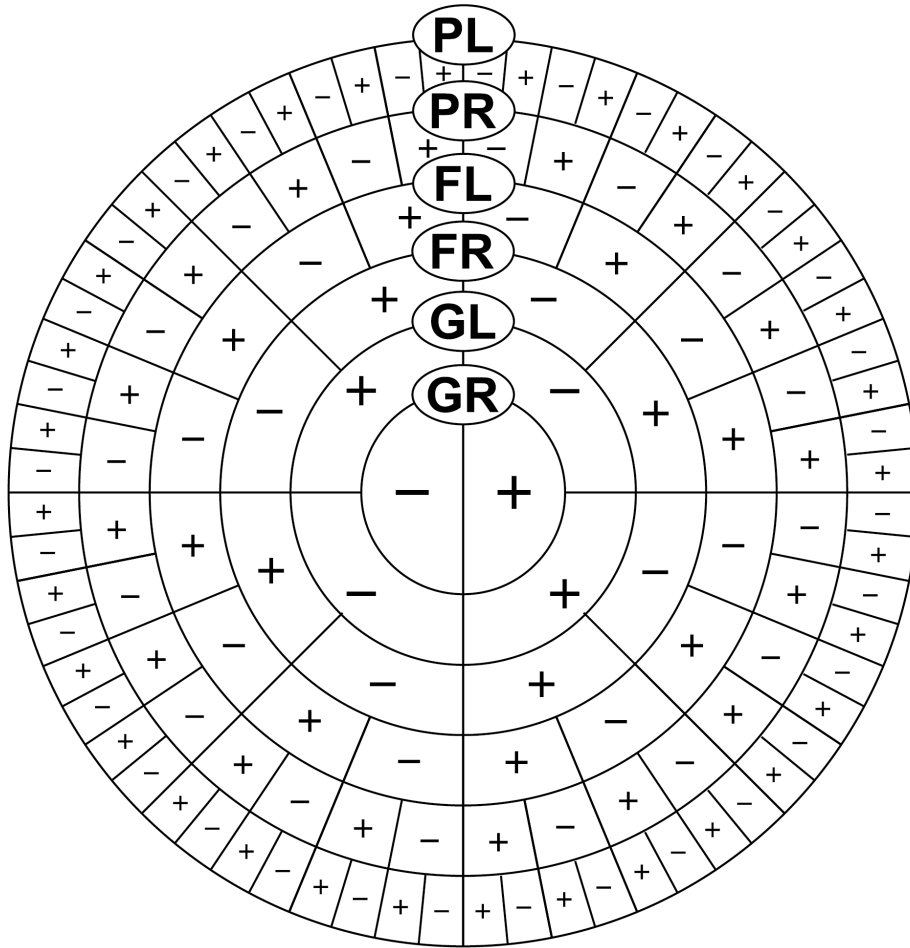
Best Restriction per Ecological Process



GRIIS
GLOBAL REGISTER OF
INTRODUCED AND INVASIVE SPECIES

Eco-Evolutionary
Rarity of 5,611
Global
Angiosperms

The Global Model of Discretized Rarity performed well across sample sizes...

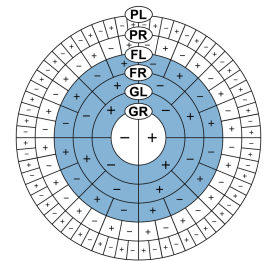
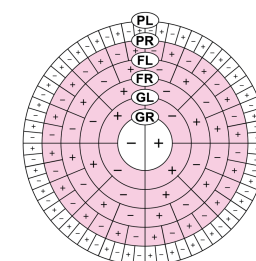
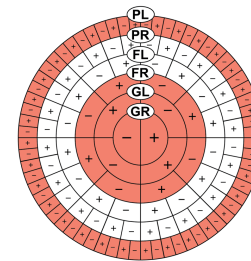
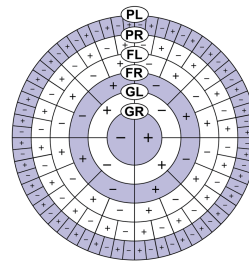
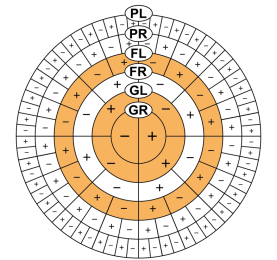
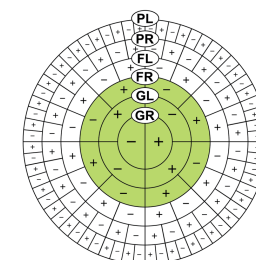
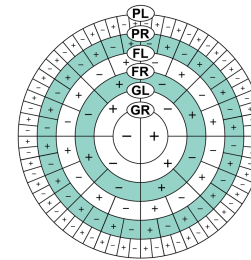
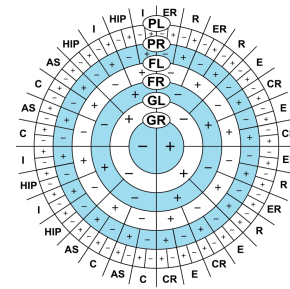
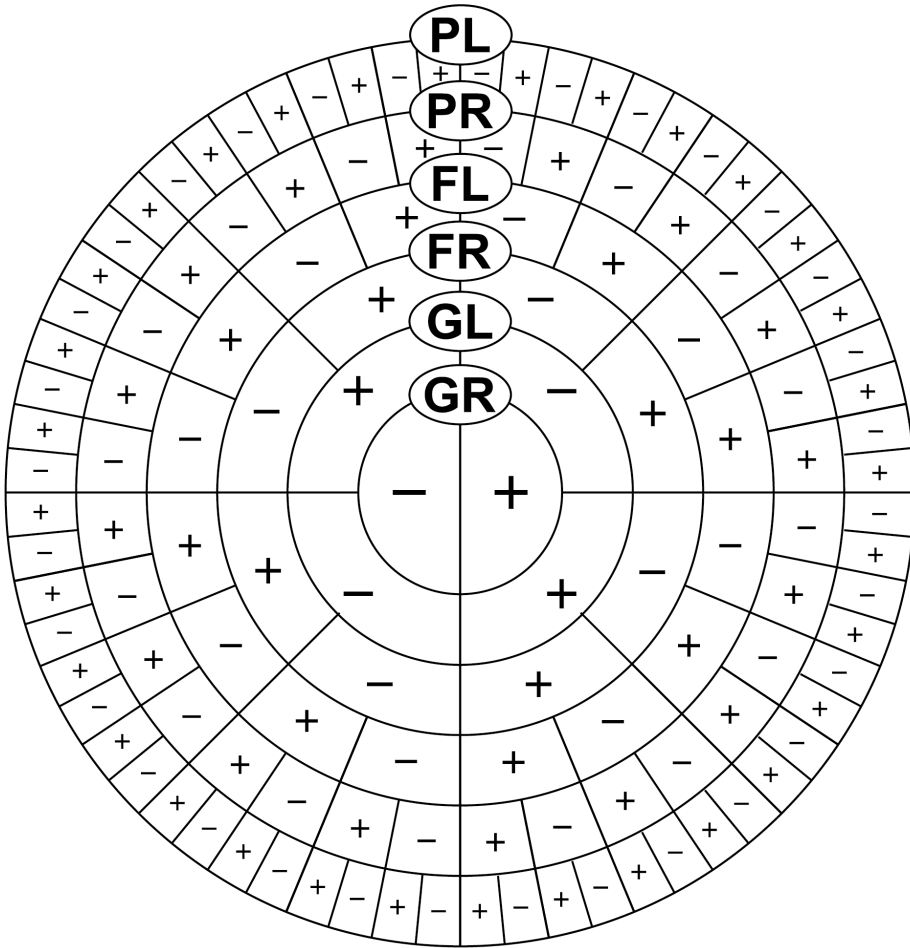


- 12.5% - 39% of variation in distribution change
- 13% - 49.1% of variation in flowering phenology

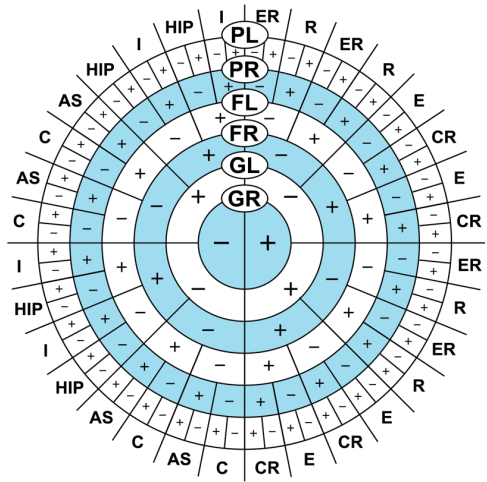
The Global Model of Discretized Rarity performed well across sample sizes...

- 12.5% - 39% of variation in distribution change
- 13% - 49.1% of variation in flowering phenology

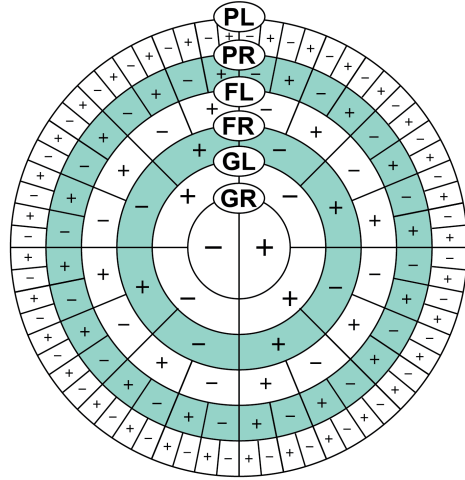
...but many restrictions outperformed the GDR



Small Sample Size



EER



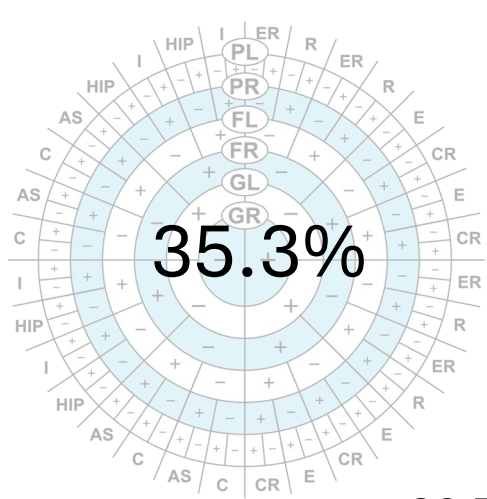
$F_R P_R$

Distribution
Change

Flowering
Phenology

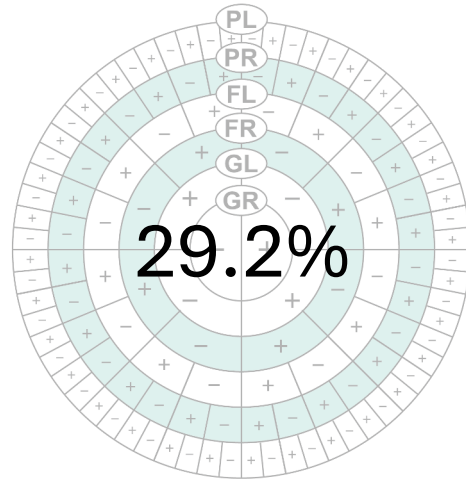
Large Sample Size

Small Sample Size



EER

36.7% AIC
Weight



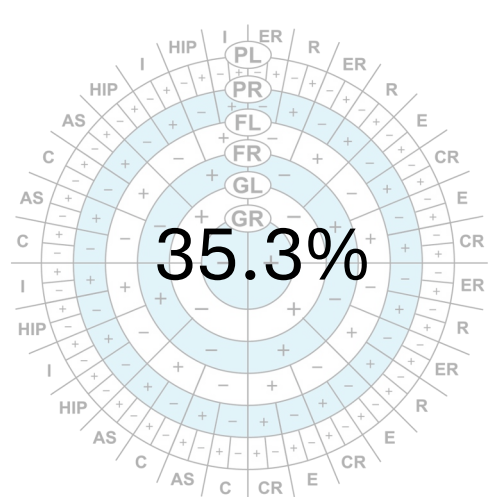
$F_R P_R$

Distribution
Change

Flowering
Phenology

Large Sample Size

Small Sample Size

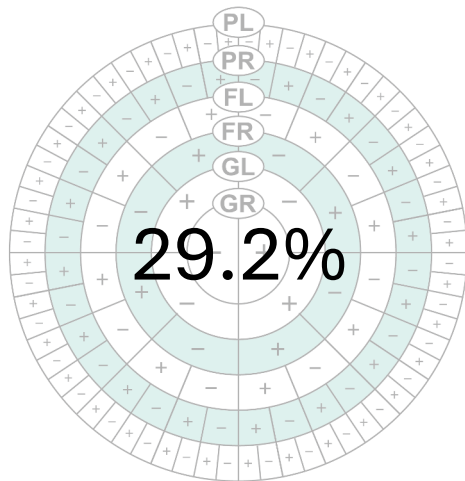


EER

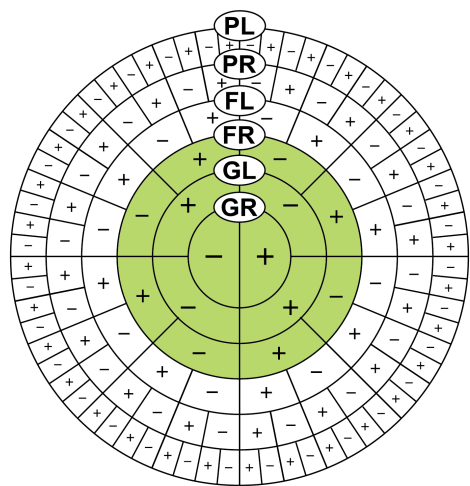
36.7% AIC
Weight

$F_R P_R$

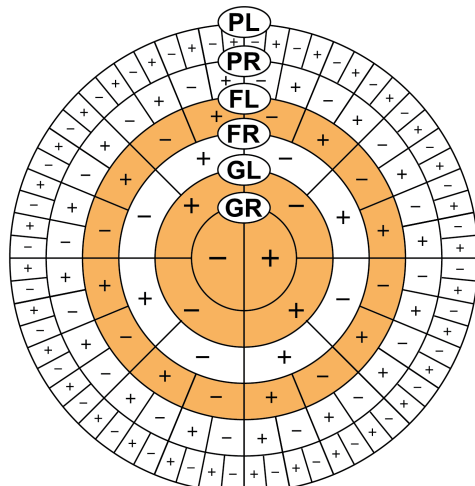
Large Sample Size



Distribution
Change



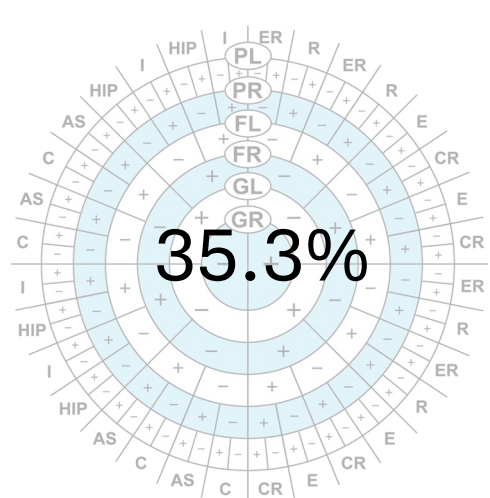
$G_{RL} F_R$



$G_{RL} F_L$

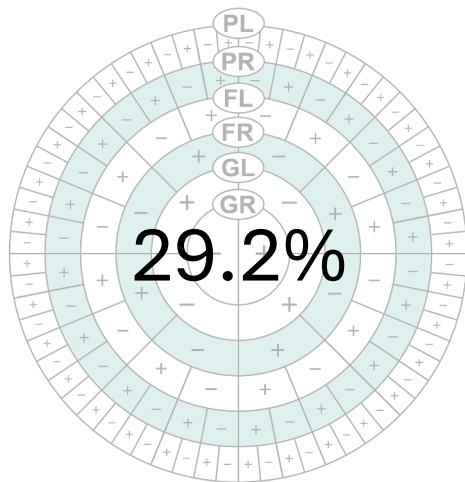
Flowering
Phenology

Small Sample Size



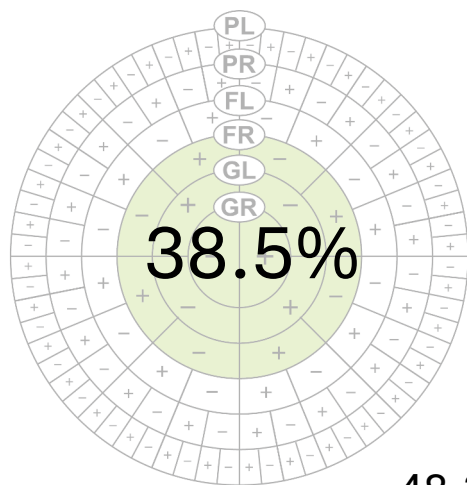
EER

36.7% AIC
Weight



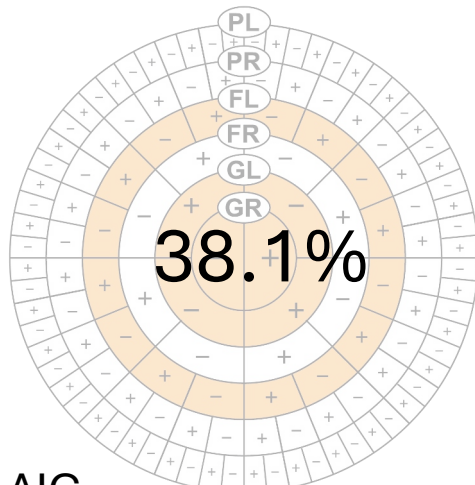
$F_R P_R$

Distribution
Change



$G_{RL} F_R$

48.3% AIC
Weight

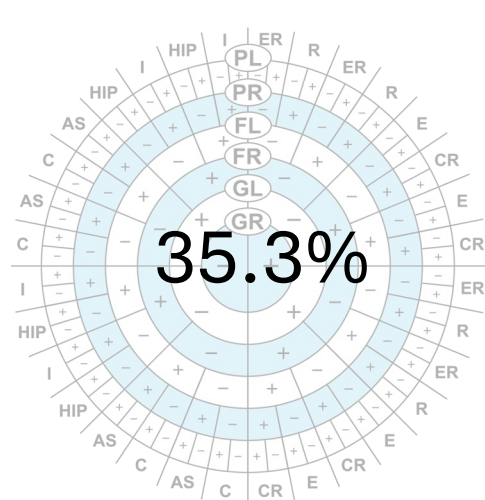


$G_{RL} F_L$

Flowering
Phenology

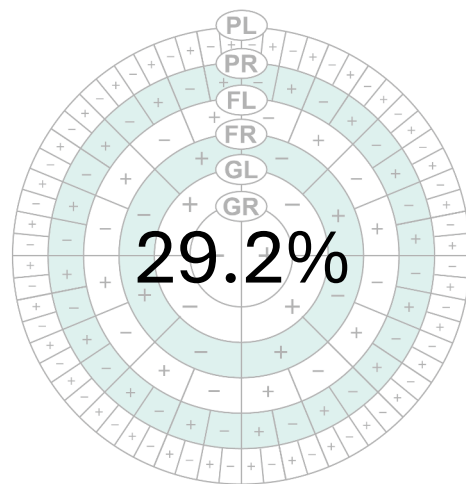
Large Sample Size

Small Sample Size

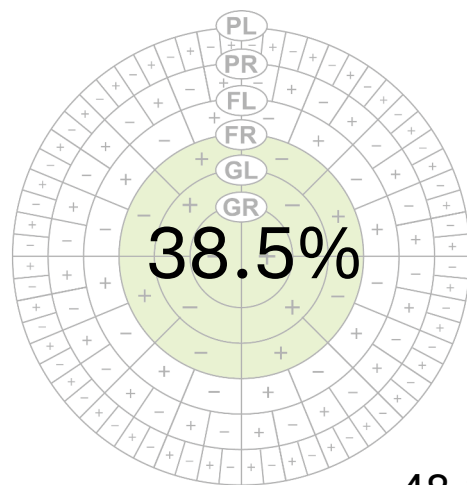


EER

36.7% AIC
Weight

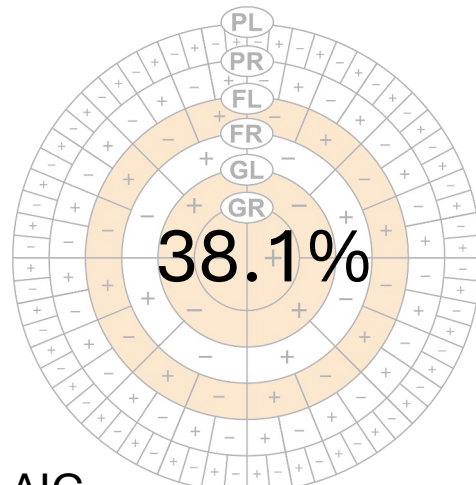


$F_R P_R$



$G_{RL} F_R$

48.3% AIC
Weight

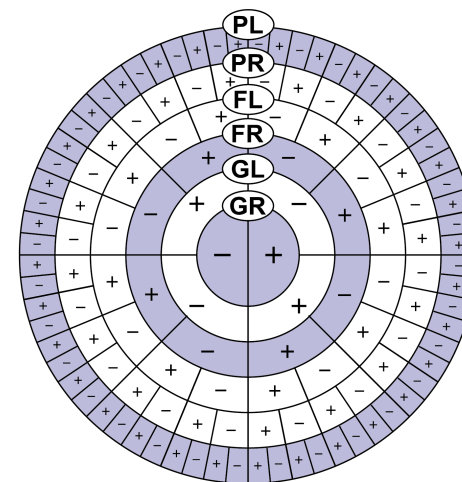


$G_{RL} F_L$

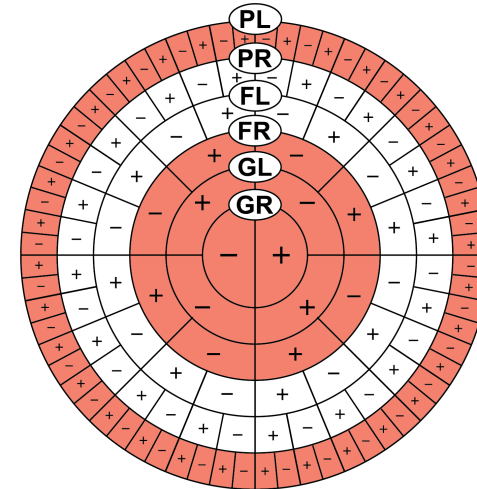
Distribution
Change

Flowering
Phenology

Large Sample Size

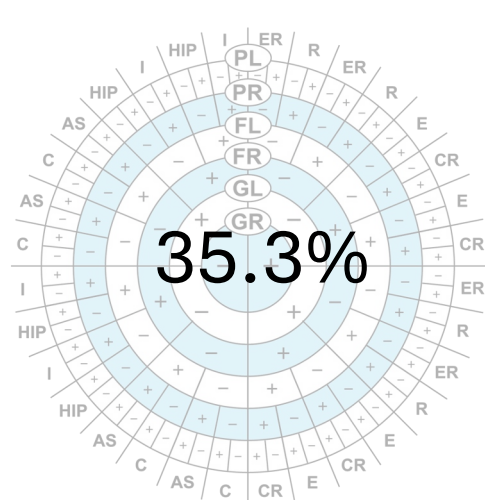


$G_R F_R P_L$



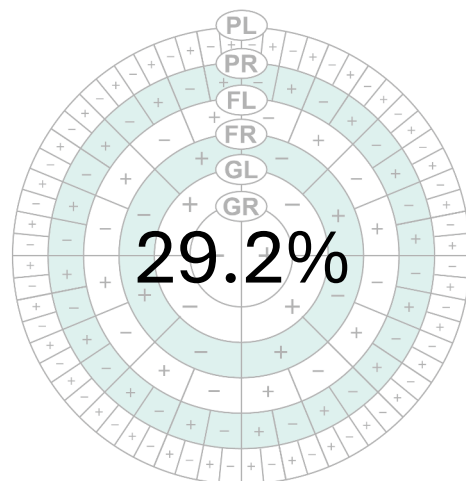
$G_{RL} F_R P_L$

Small Sample Size

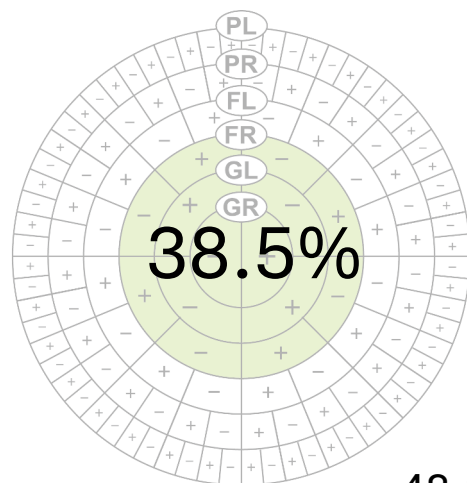


EER

36.7% AIC
Weight

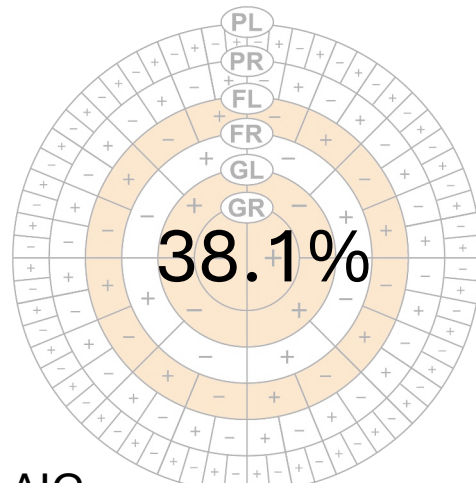


$F_R P_R$



$G_{RL} F_R$

48.3% AIC
Weight

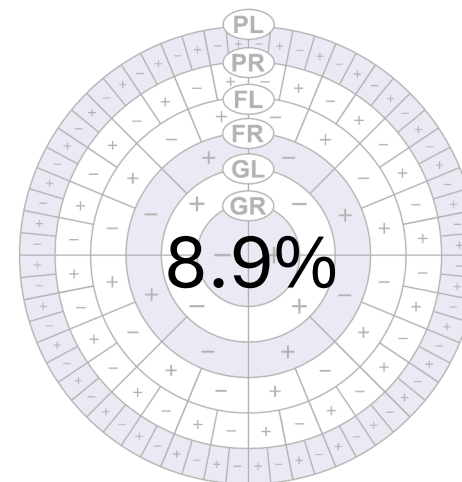


$G_{RL} F_L$

Distribution
Change

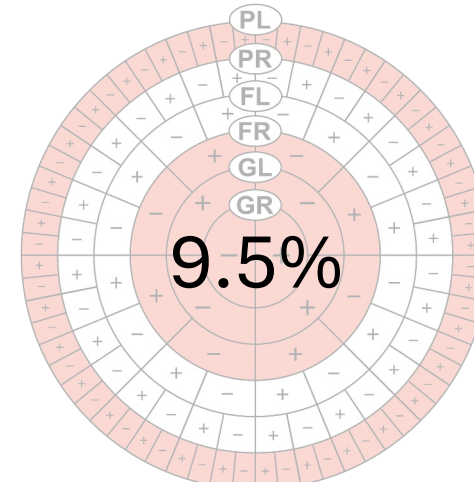
Flowering
Phenology

Large Sample Size



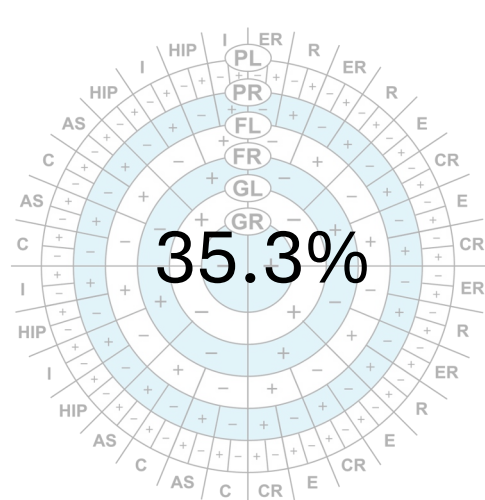
$G_R F_R P_L$

89.3% AIC
Weight



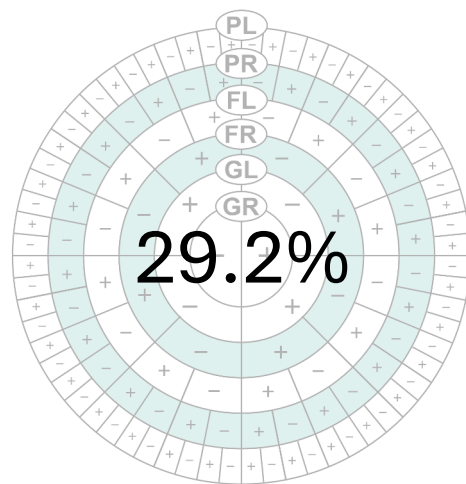
$G_{RL} F_R P_L$

Small Sample Size



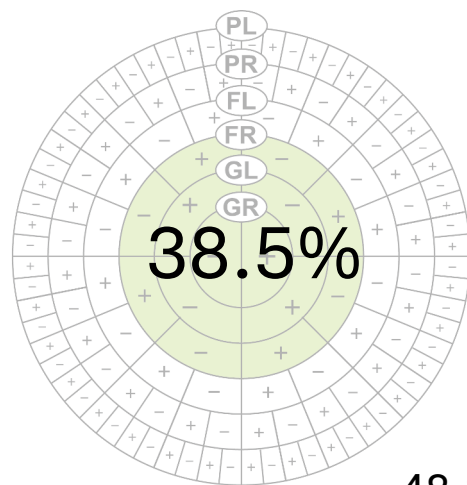
EER

36.7% AIC
Weight



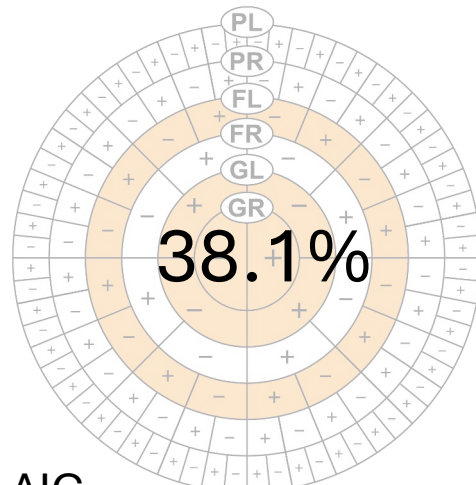
$F_R P_R$

Distribution
Change



$G_{RL} F_R$

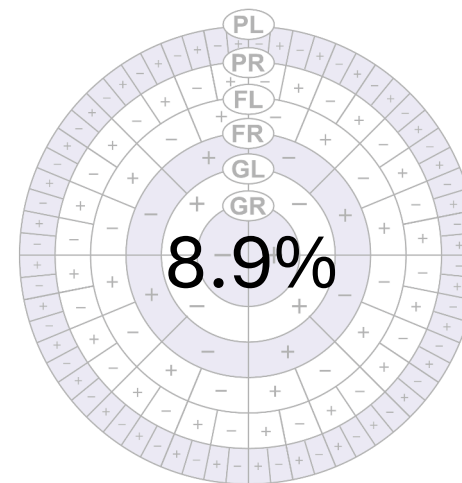
48.3% AIC
Weight



$G_{RL} F_L$

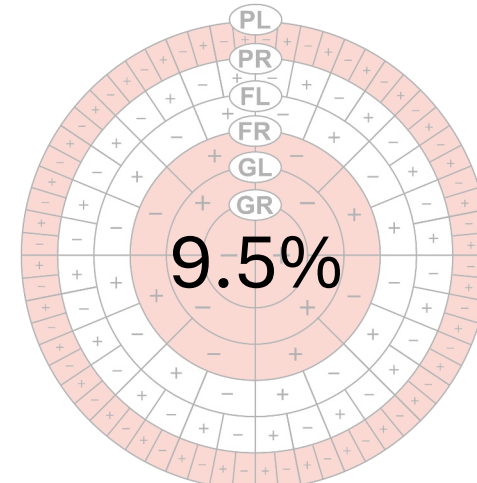
Flowering
Phenology

Large Sample Size

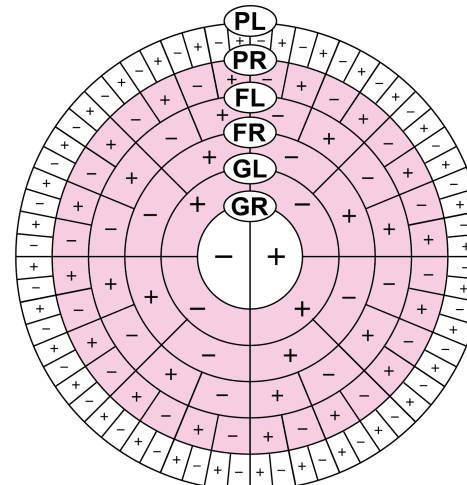


$G_R F_R P_L$

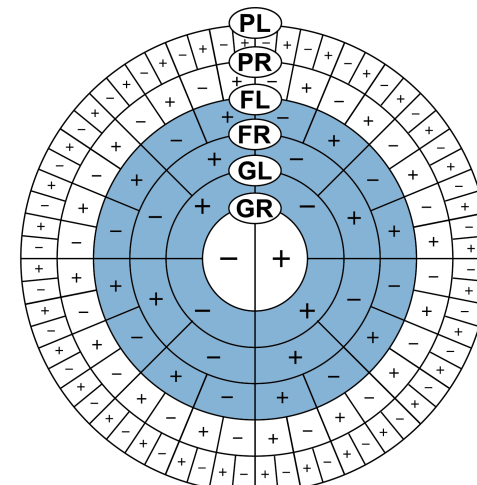
89.3% AIC
Weight



$G_{RL} F_R P_L$

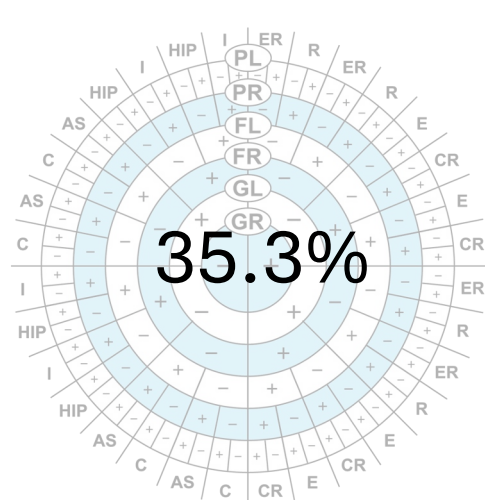


$G_L F_{RL} P_R$



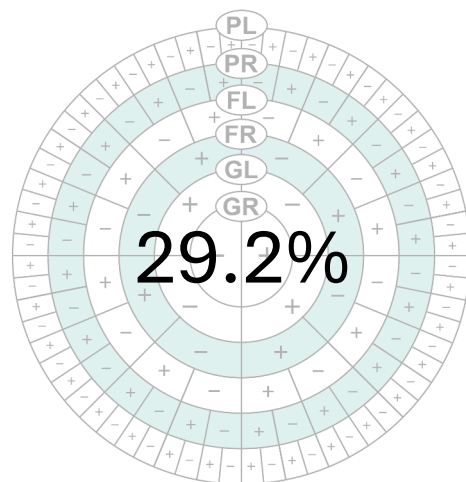
$G_L F_{RL}$

Small Sample Size

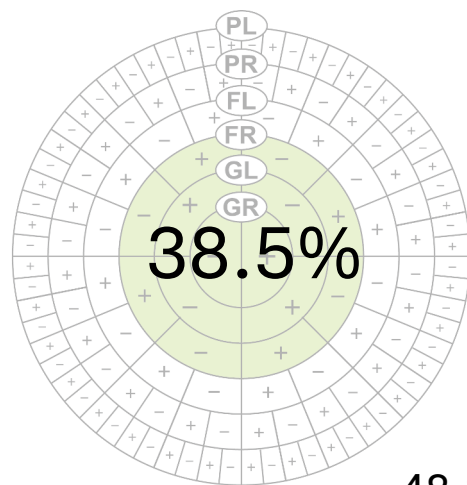


EER

36.7% AIC
Weight

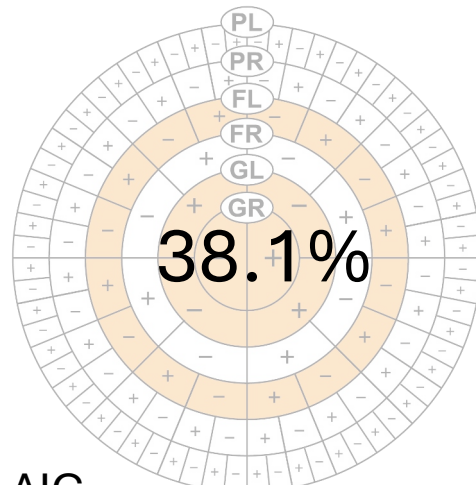


$F_R P_R$



$G_{RL} F_R$

48.3% AIC
Weight

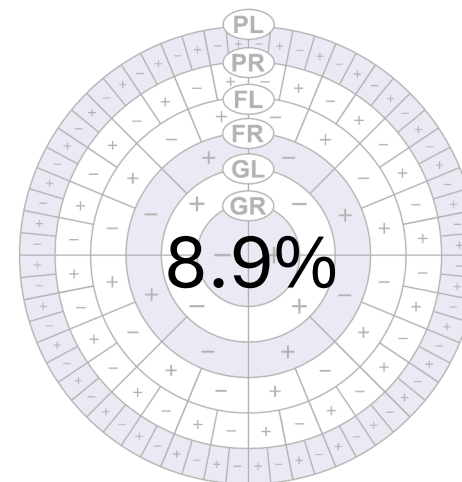


$G_{RL} F_L$

Distribution
Change

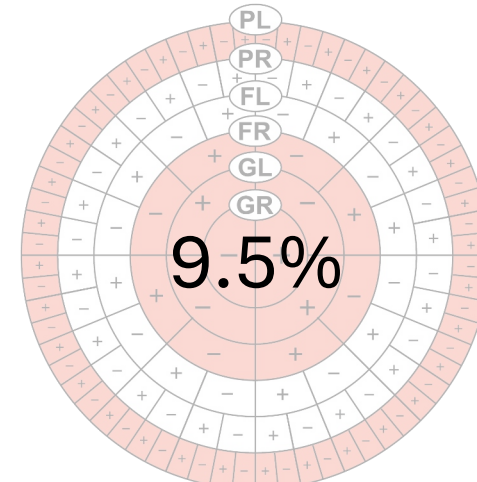
Flowering
Phenology

Large Sample Size

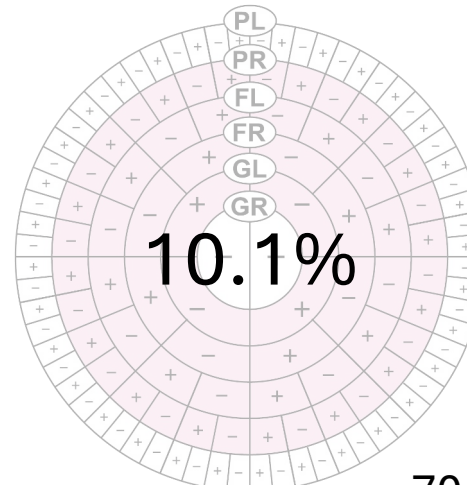


$G_R F_R P_L$

89.3% AIC
Weight

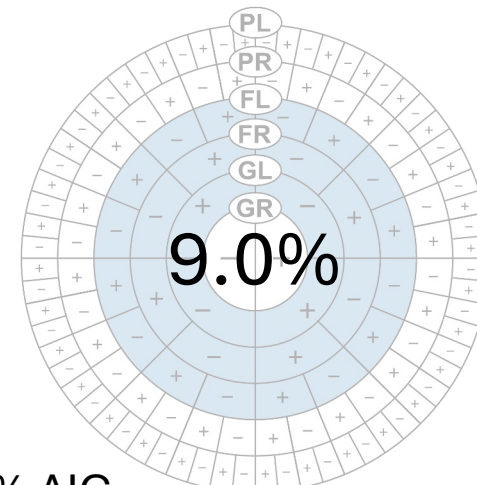


$G_{RL} F_R P_L$



$G_L F_{RL} P_R$

70.9% AIC
Weight



$G_L F_{RL}$

Small Sample Size

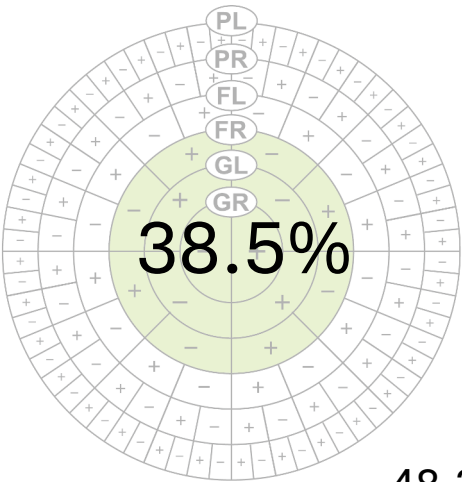
Dimension	AIC Importance
PR	96.7%
FR	91.6%
GR	73.1%
FL	23.8%
PL	15.7%
GL	15.6%

Distribution
Change

Large Sample Size

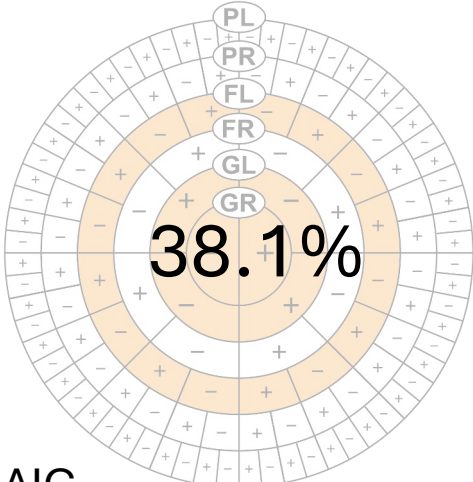
Dimension	AIC Importance
GR	100%
FR	99.8%
PL	99.3%
GL	16.6%
PR	9.2%
FL	1.1%

Flowering
Phenology

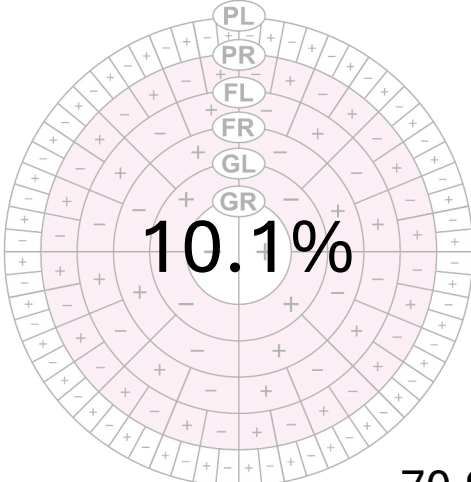


$G_{RL}F_R$

48.3% AIC
Weight

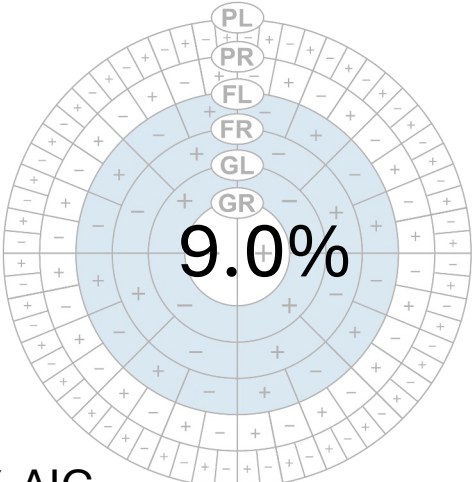


$G_{RL}F_L$



$G_LF_{RL}P_R$

70.9% AIC
Weight



G_LF_{RL}

Small Sample Size

Dimension	AIC Importance
PR	96.7%
FR	91.6%
GR	73.1%
FL	23.8%
PL	15.7%
GL	15.6%

Dimension	AIC Importance
GR	100%
GL	100%
FR	69.8%
FL	45.1%
PR	28.7%
PL	18.5%

Distribution
Change

Flowering
Phenology

Large Sample Size

Dimension	AIC Importance
GR	100%
FR	99.8%
PL	99.3%
GL	16.6%
PR	9.2%
FL	1.1%

Dimension	AIC Importance
GL	100%
FR	100%
FL	99.6%
PR	56.3%
PL	23.9%
GR	8.0%

Small Sample Size

Dimension	AIC Importance
PR	96.7%
FR	91.6%
GR	73.1%
FL	23.8%
PL	15.7%
GL	15.6%

Dimension	AIC Importance
GR	100%
GL	100%
FR	69.8%
FL	45.1%
PR	28.7%
PL	18.5%

Distribution
Change

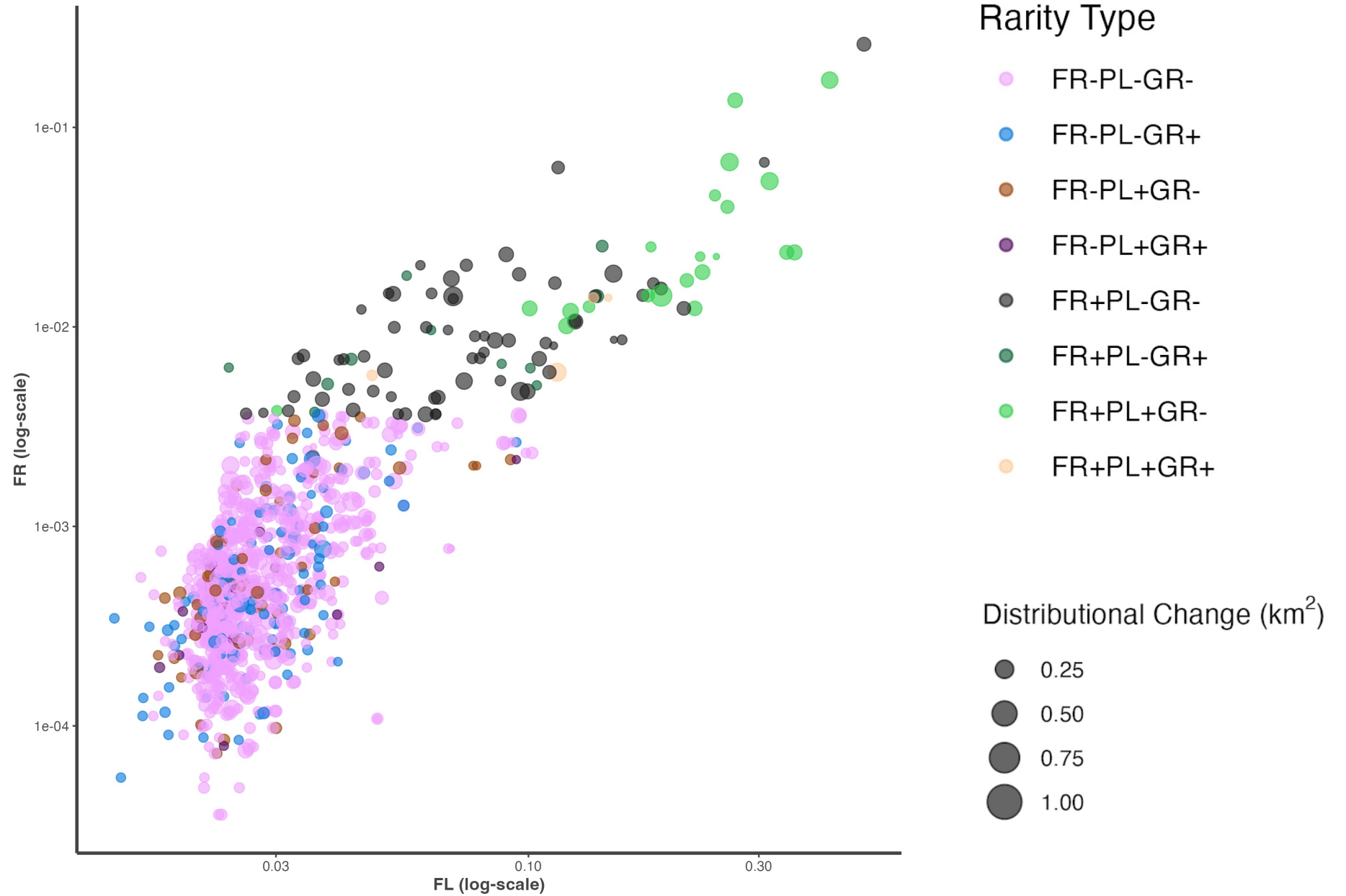
Flowering
Phenology

Large Sample Size

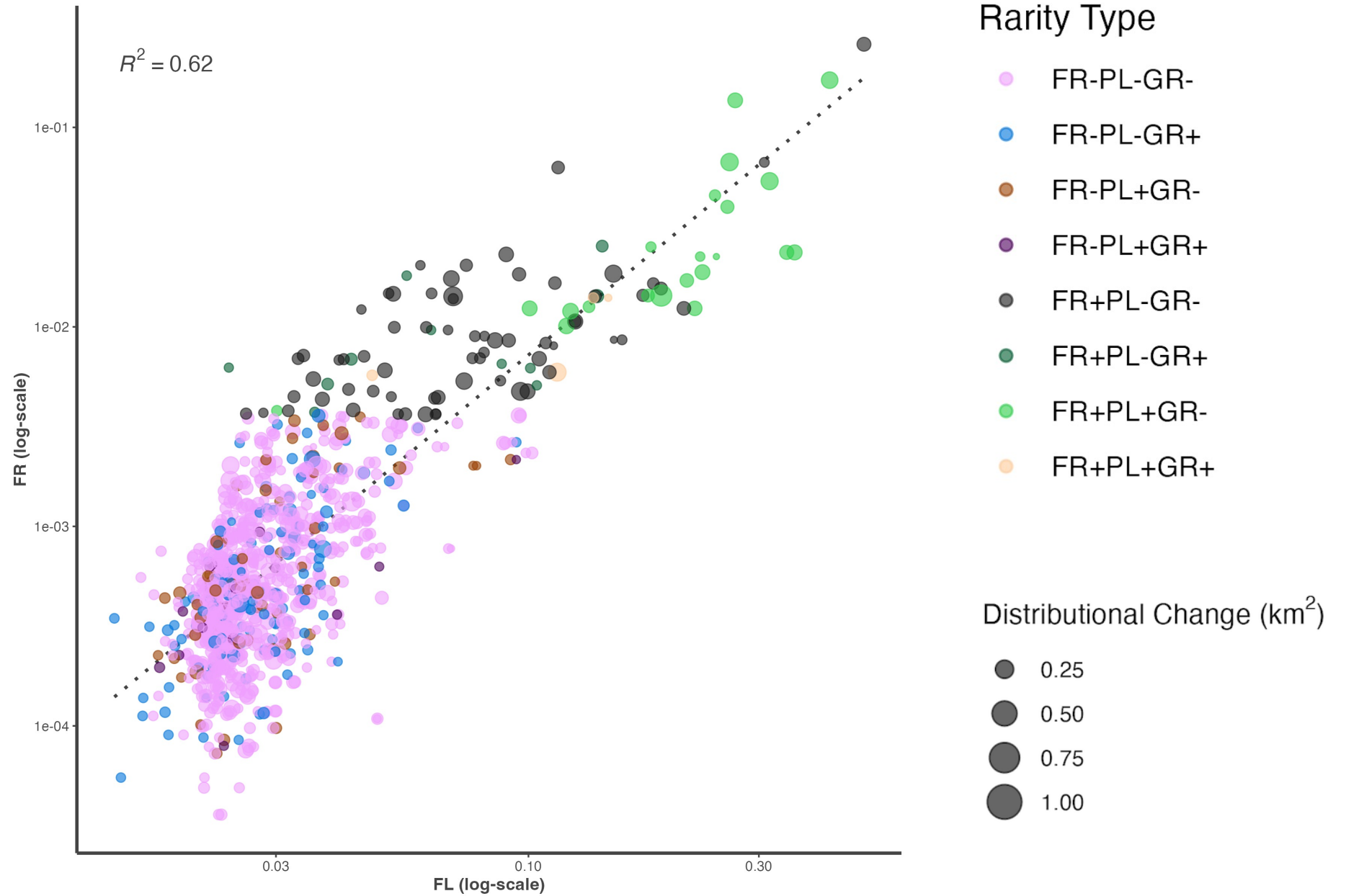
Dimension	AIC Importance
GR	100%
FR	99.8%
PL	99.3%
GL	16.6%
PR	9.2%
FL	1.1%

Dimension	AIC Importance
GL	100%
FR	100%
FL	99.6%
PR	56.3%
PL	23.9%
GR	8.0%

Distribution Change

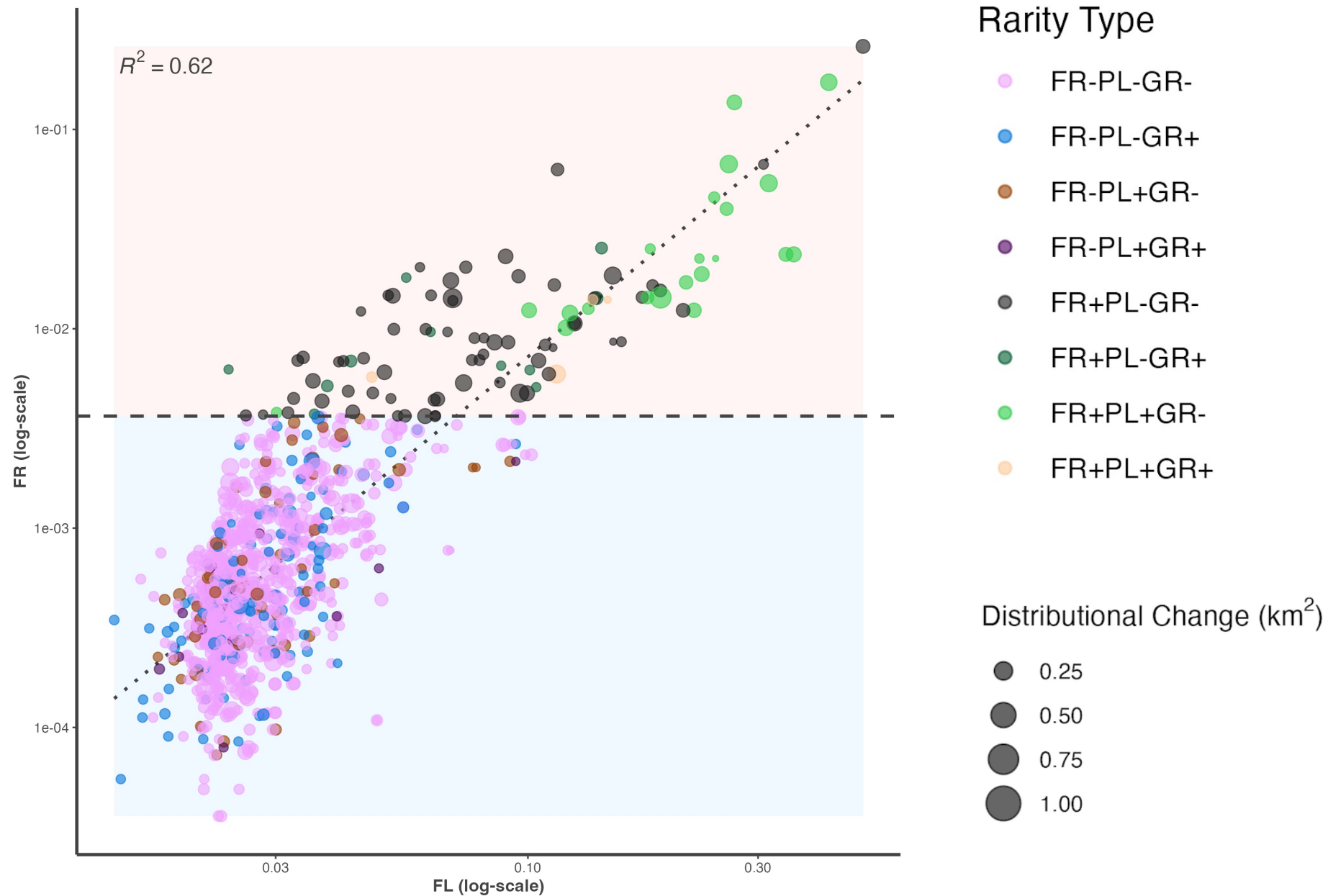


Distribution Change



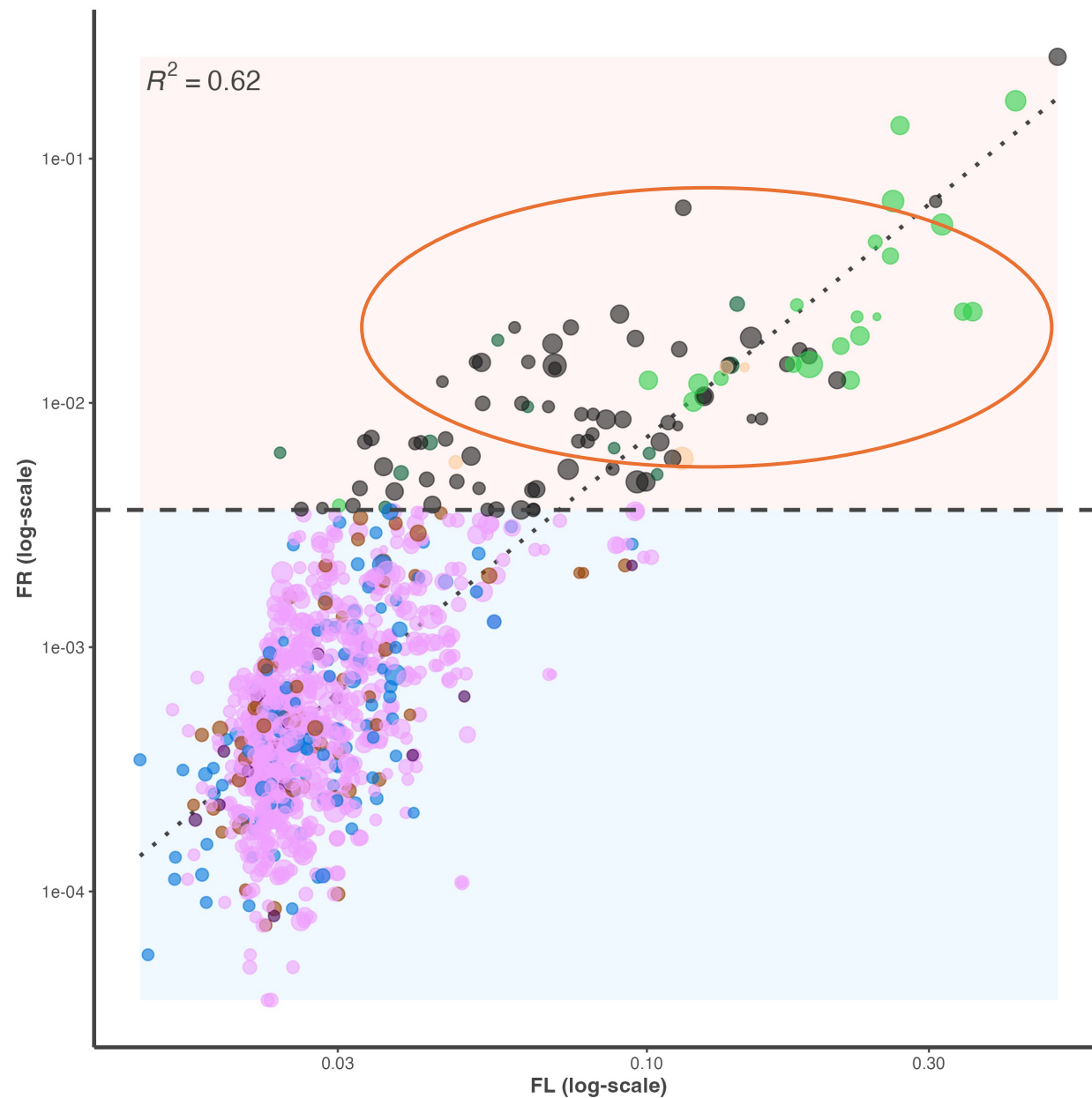
Distribution Change

$G_R F_R P_L$



Distribution Change

$G_R F_R P_L$



Rarity Type

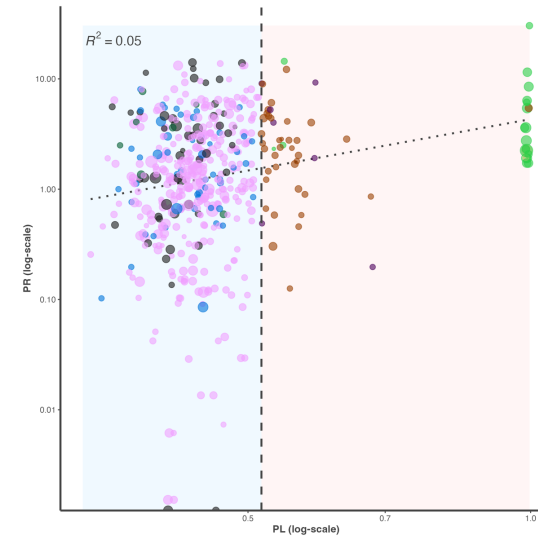
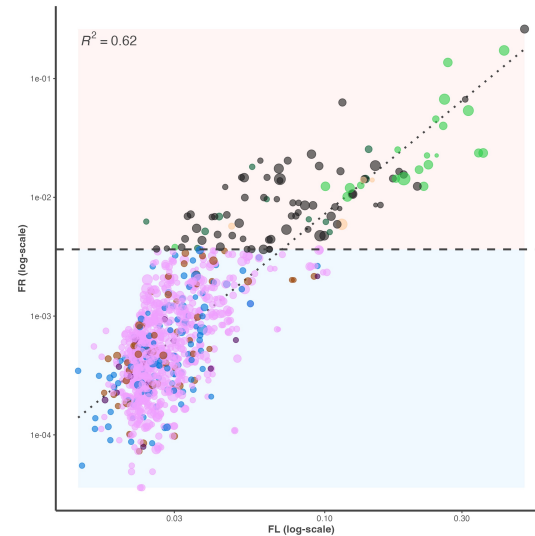
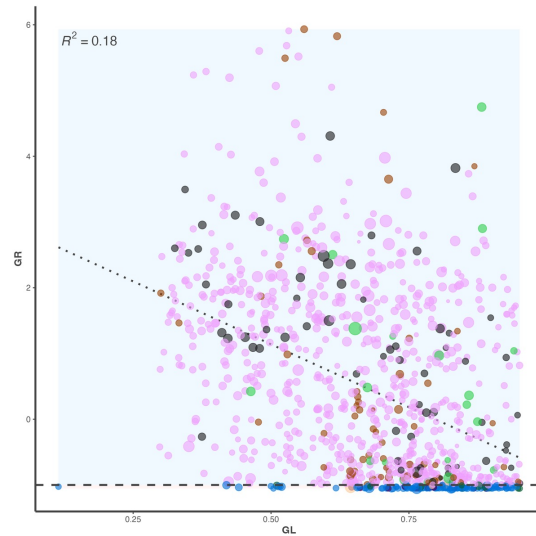
- FR-PL-GR-
- FR-PL-GR+
- FR-PL+GR-
- FR-PL+GR+
- FR+PL-GR-
- FR+PL-GR+
- FR+PL+GR-
- FR+PL+GR+

Distributional Change (km^2)

- 0.25
- 0.50
- 0.75
- 1.00

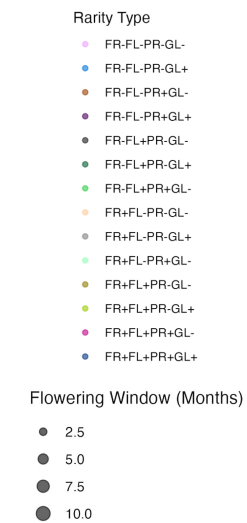
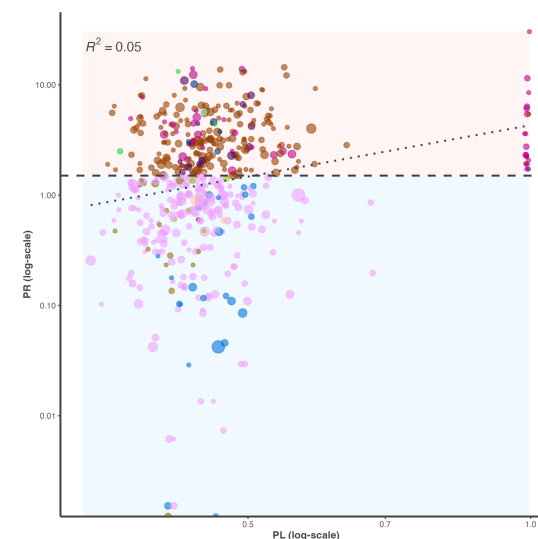
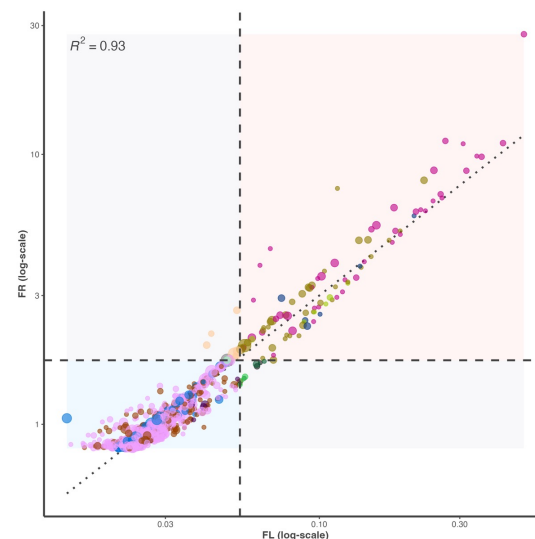
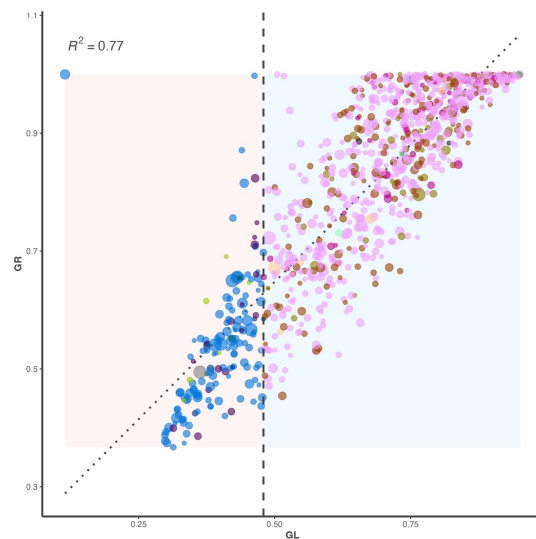
Distribution Change

$G_R F_R P_L$



Flowering Phenology

$G_L F_{RL} P_R$



EER of Global Angiosperms

Range Size

Large

Small

**Functional
Distinctiveness**

Distinct

Indistinct

Distinct

Indistinct

**Evolutionary
Distinctiveness**

Distinct

Indistinct

Indicator (0.9%)	Adaptable Survivor (7.4%)	Classically Rare (0.4%)	Relict (1.3%)
High Invasive Potential (6.3%)	Common (70.4%)	Endemic (2.4%)	Environmentally Rare (10.9%)

Invasiveness of EER Types

Range Size

Large

Small

**Functional
Distinctiveness**

Distinct

Indistinct

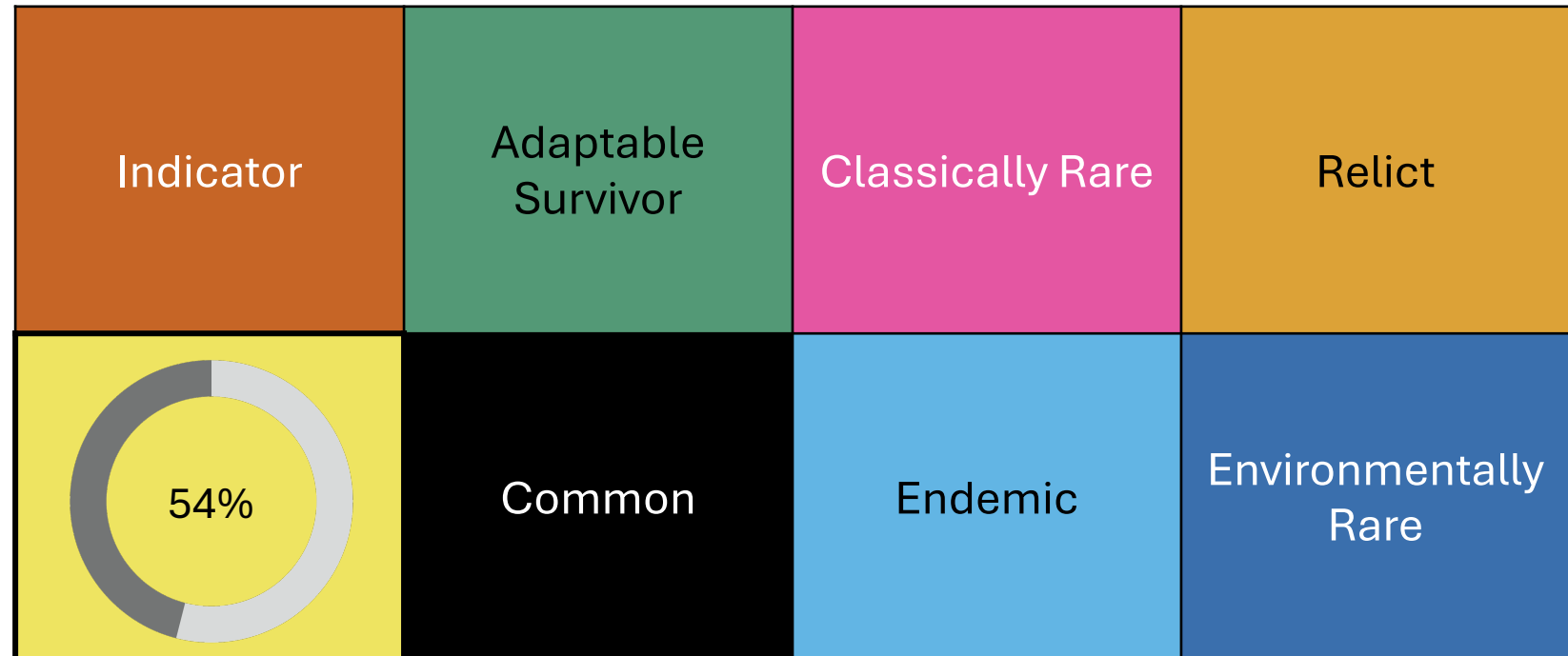
Distinct

Indistinct

**Evolutionary
Distinctiveness**

Distinct

Indistinct



Indicator
Eucalyptus exserta



Sun et al. 2016

Adaptable Survivor
Hibbertia riparia



Hammer et al. 2025

Classically Rare
Paulownia kawakamii



Tony Kirkham

Relict
Gnetum gnemon



Ickert-Bond et al. 2016

**High Invasive
Potential**
Pinus radiata



Eugene Zelenko

Williams & Wardle 2007

Common
Acer rubrum



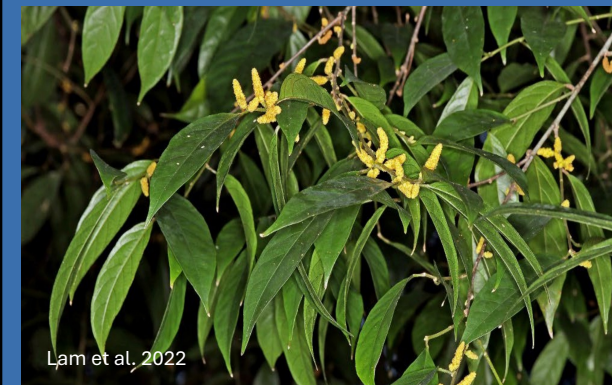
Endemic
Flindersia brayleyana



Margaret Donald

Boland 2006

**Environmentally
Rare**
Aporosa frutescens



Lam et al. 2022

1

Can we use rarity as a
tool to understand
ecological processes?
How?

2

Are rarity types robust
and ecologically
relevant?

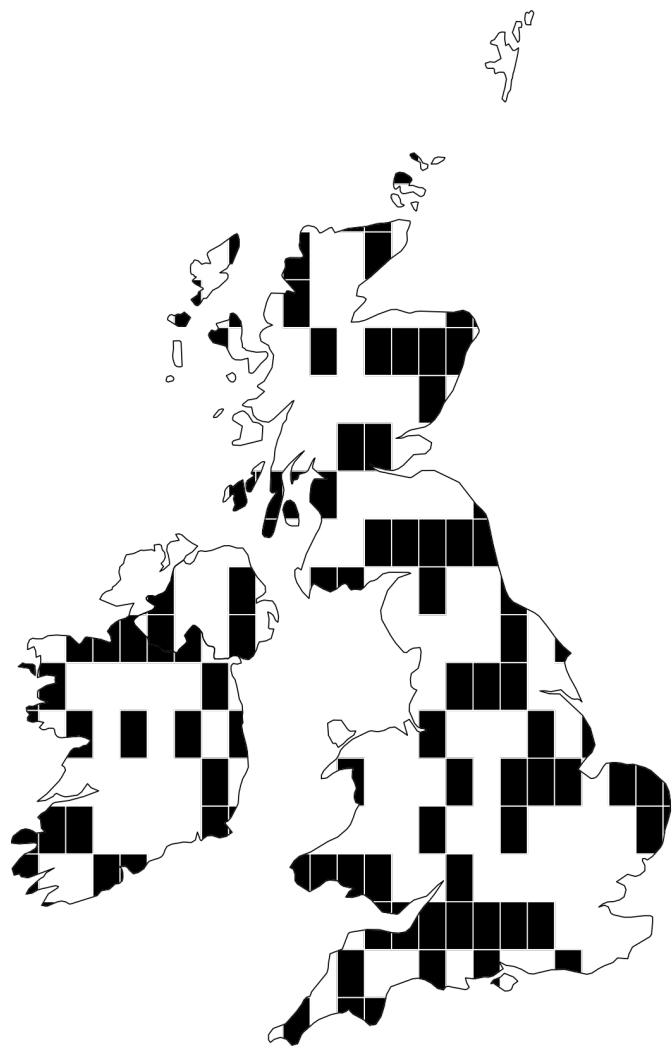
1

Can we use rarity as a
tool to understand
ecological processes?
How?

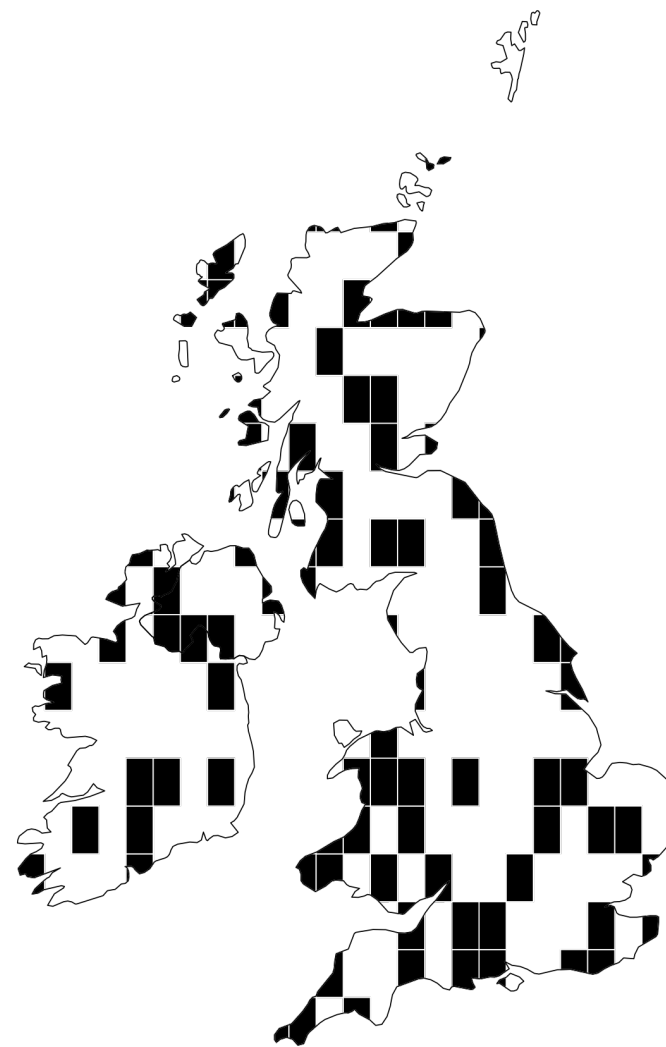
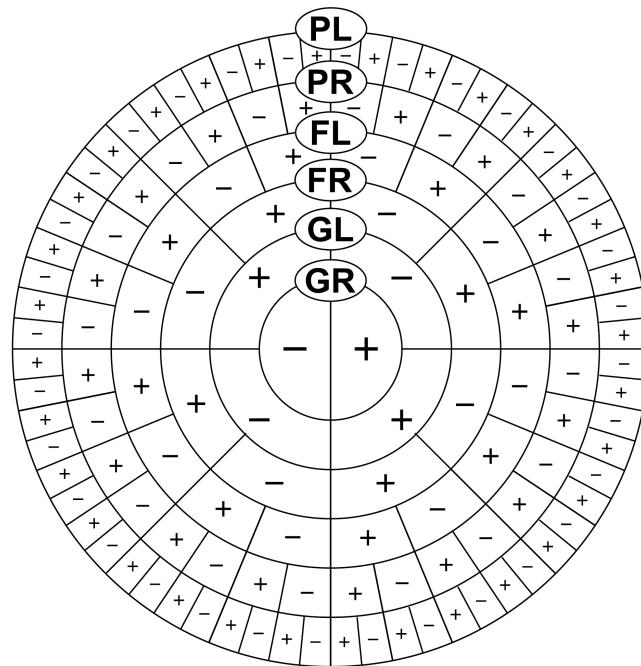
**Yes... with some
caveats**

2

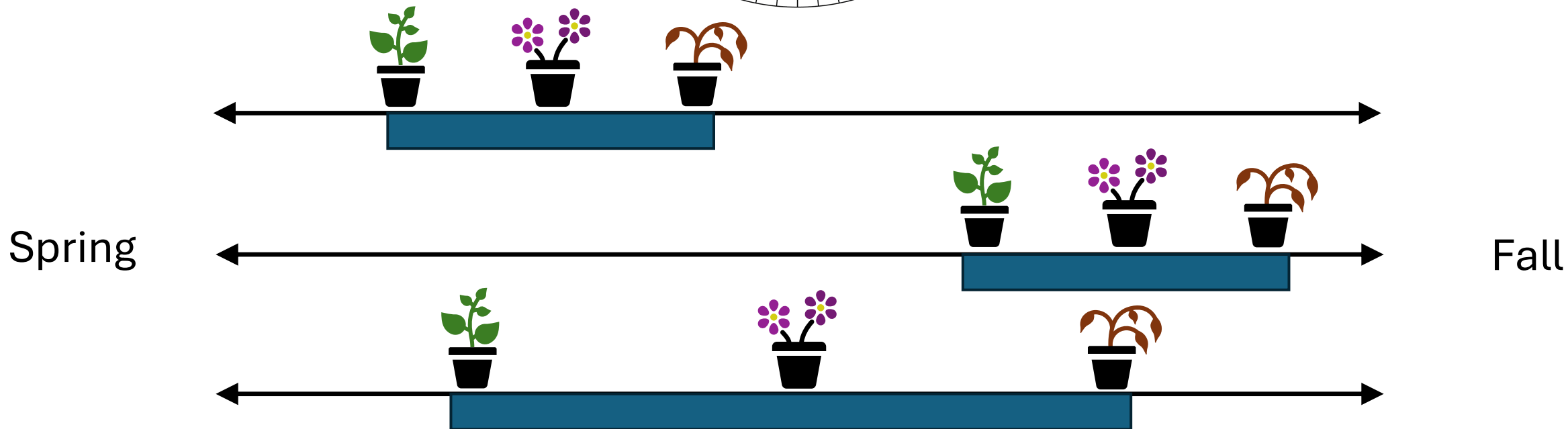
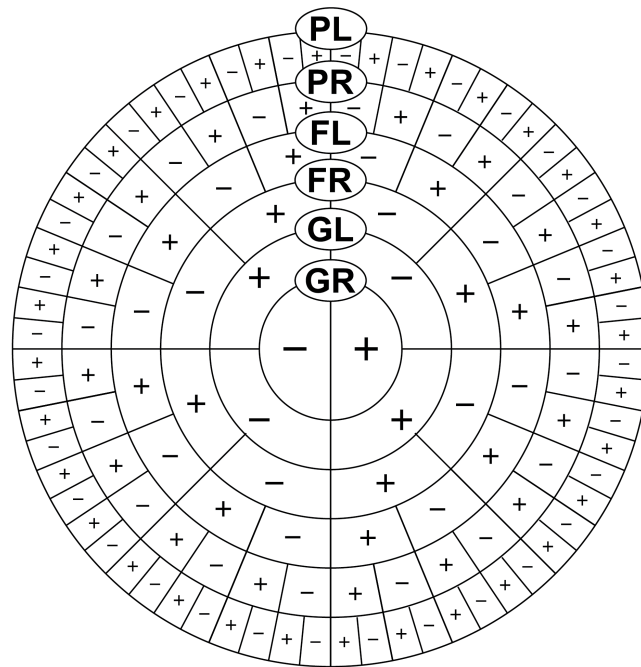
Are rarity types robust
and ecologically
relevant?

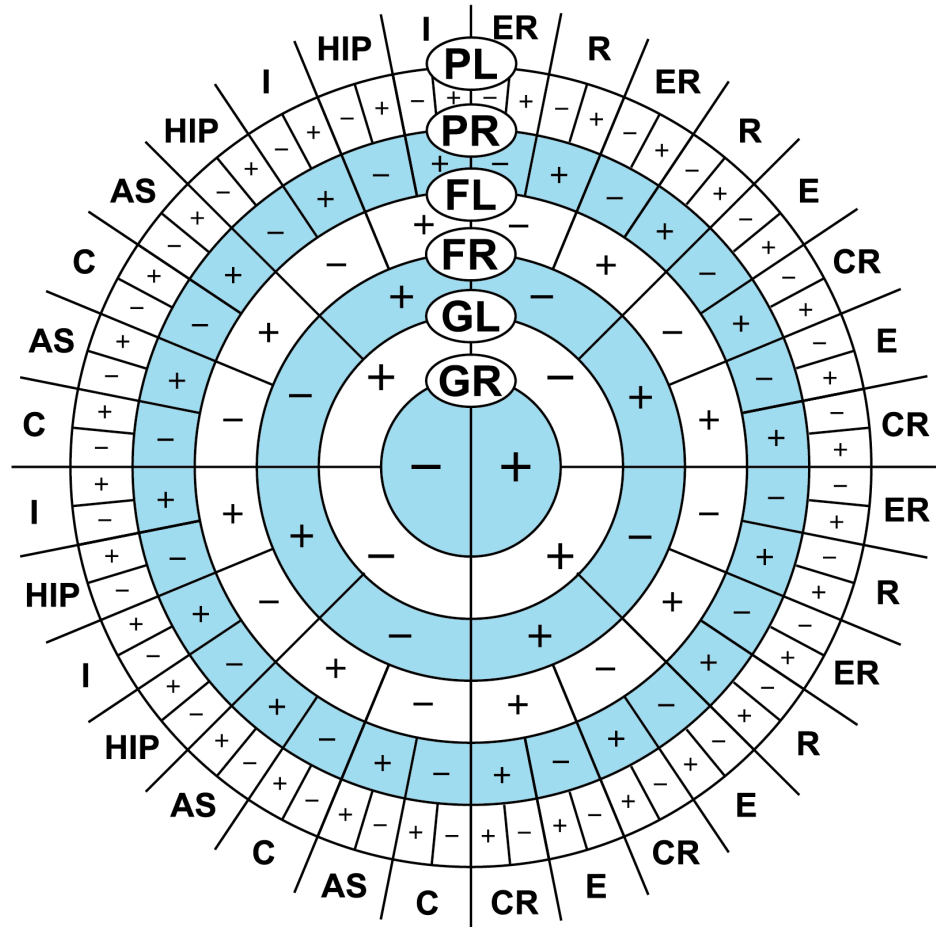


1987



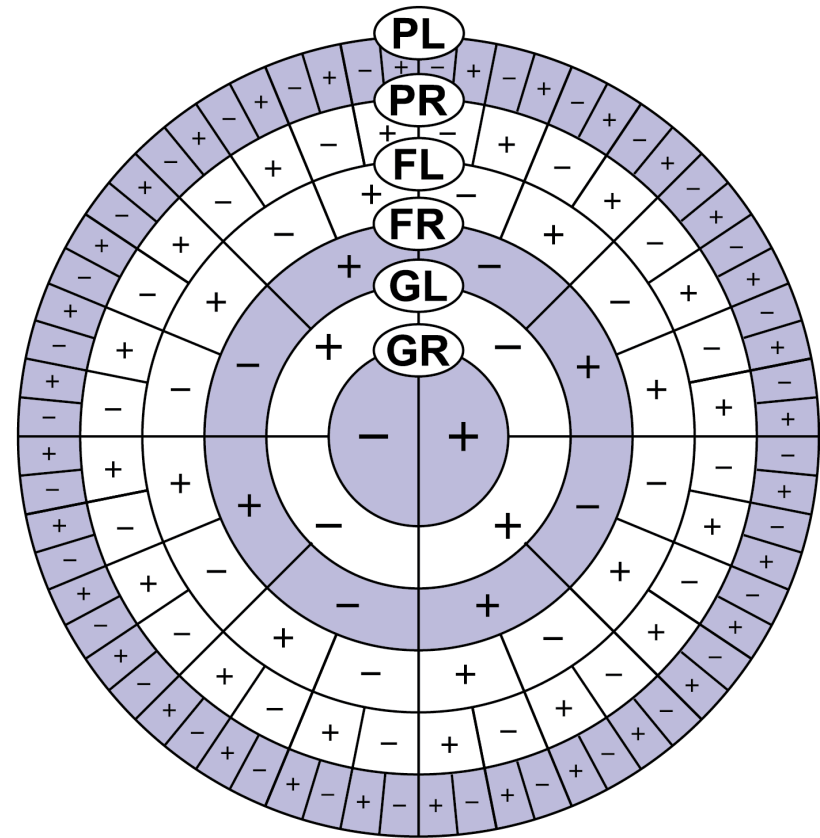
2019

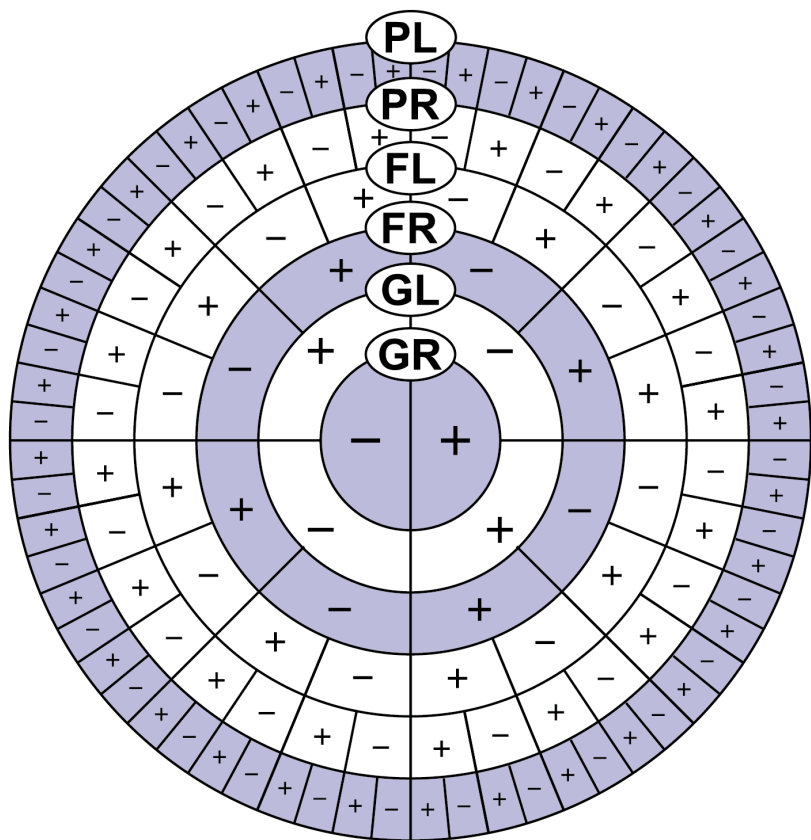




Eco-Evolutionary Rarity

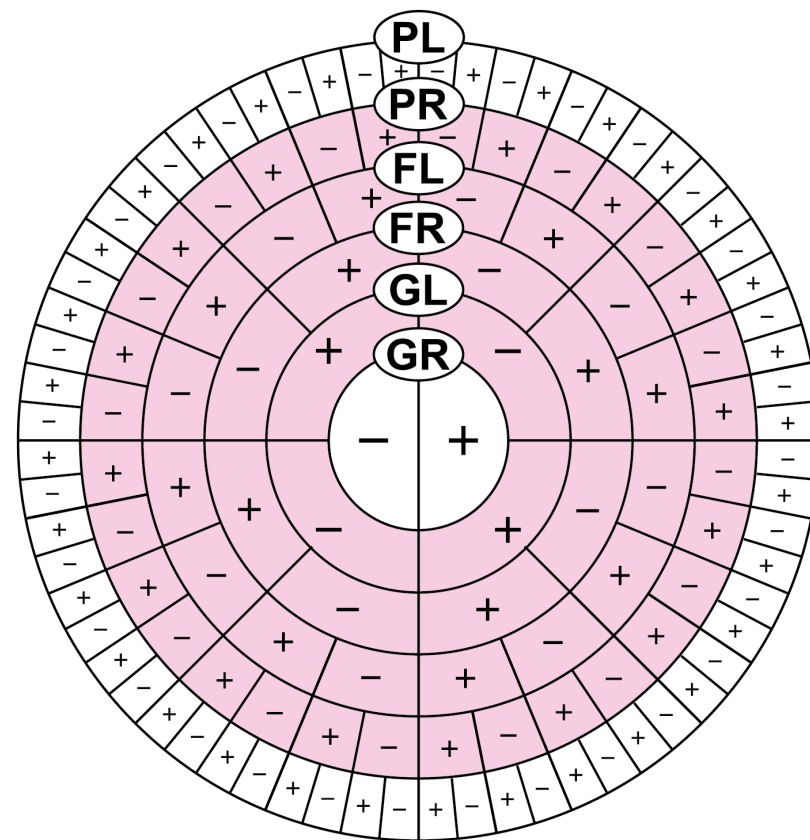
VS


$$G_R F_R P_L$$

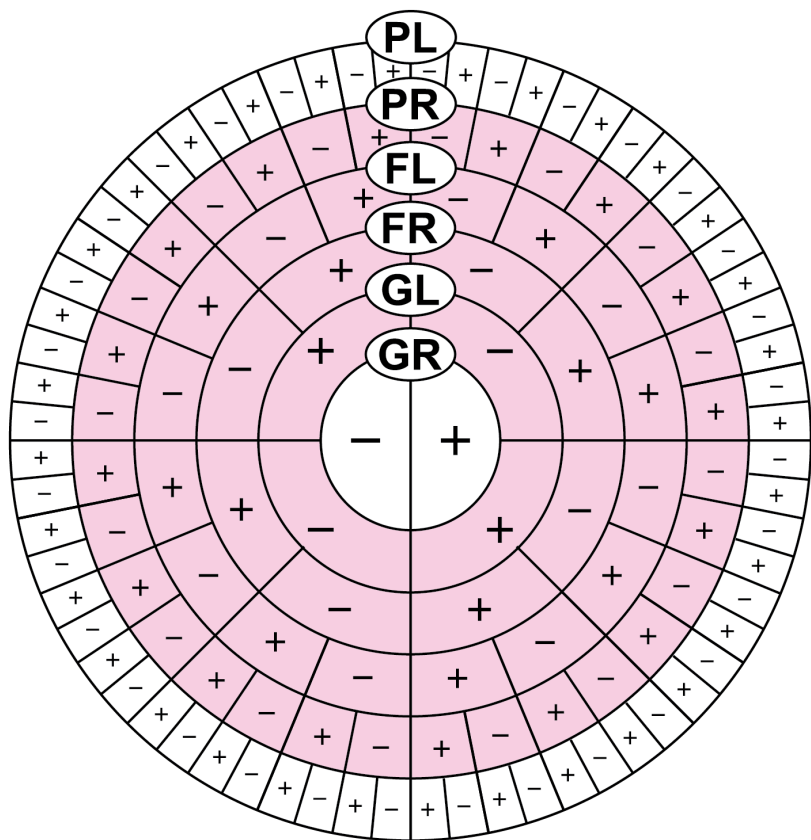


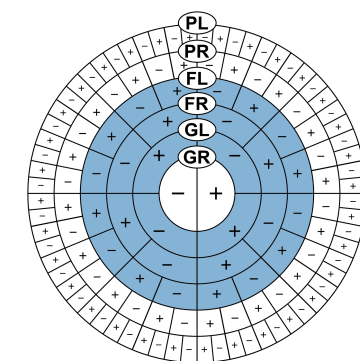
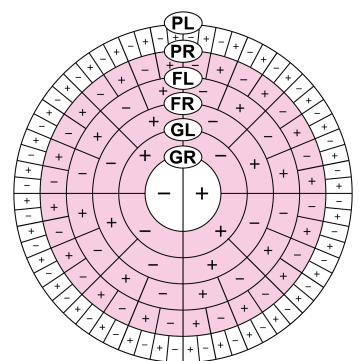
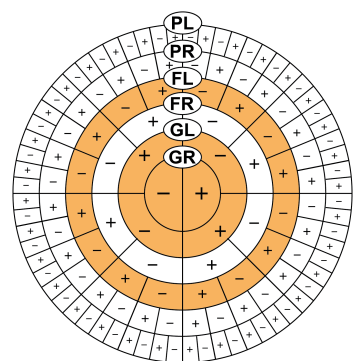
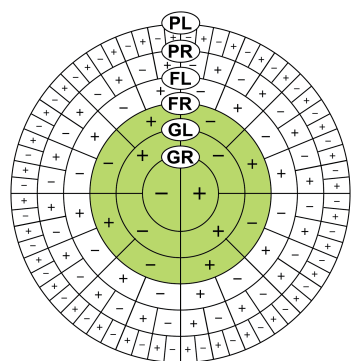
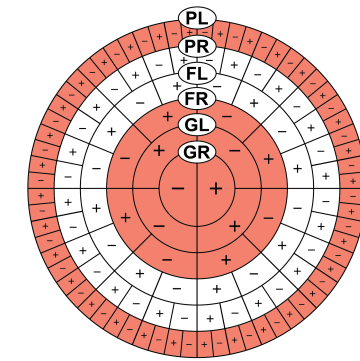
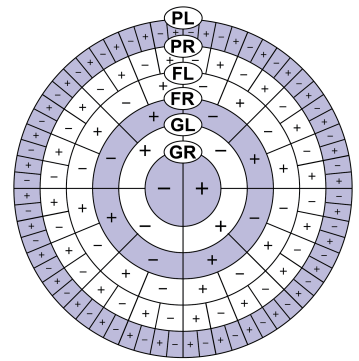
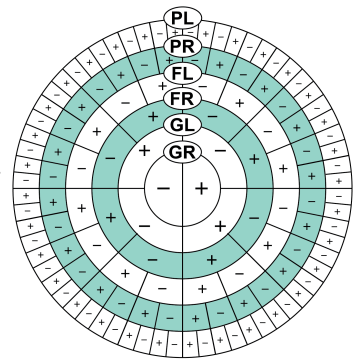
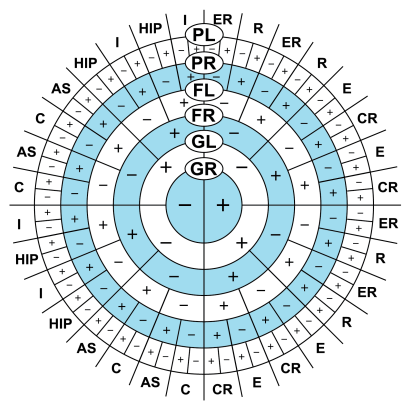
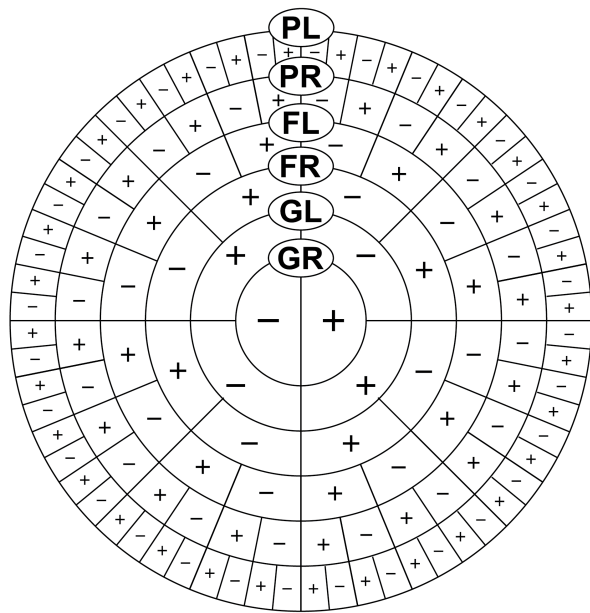
$G_R F_R P_L$

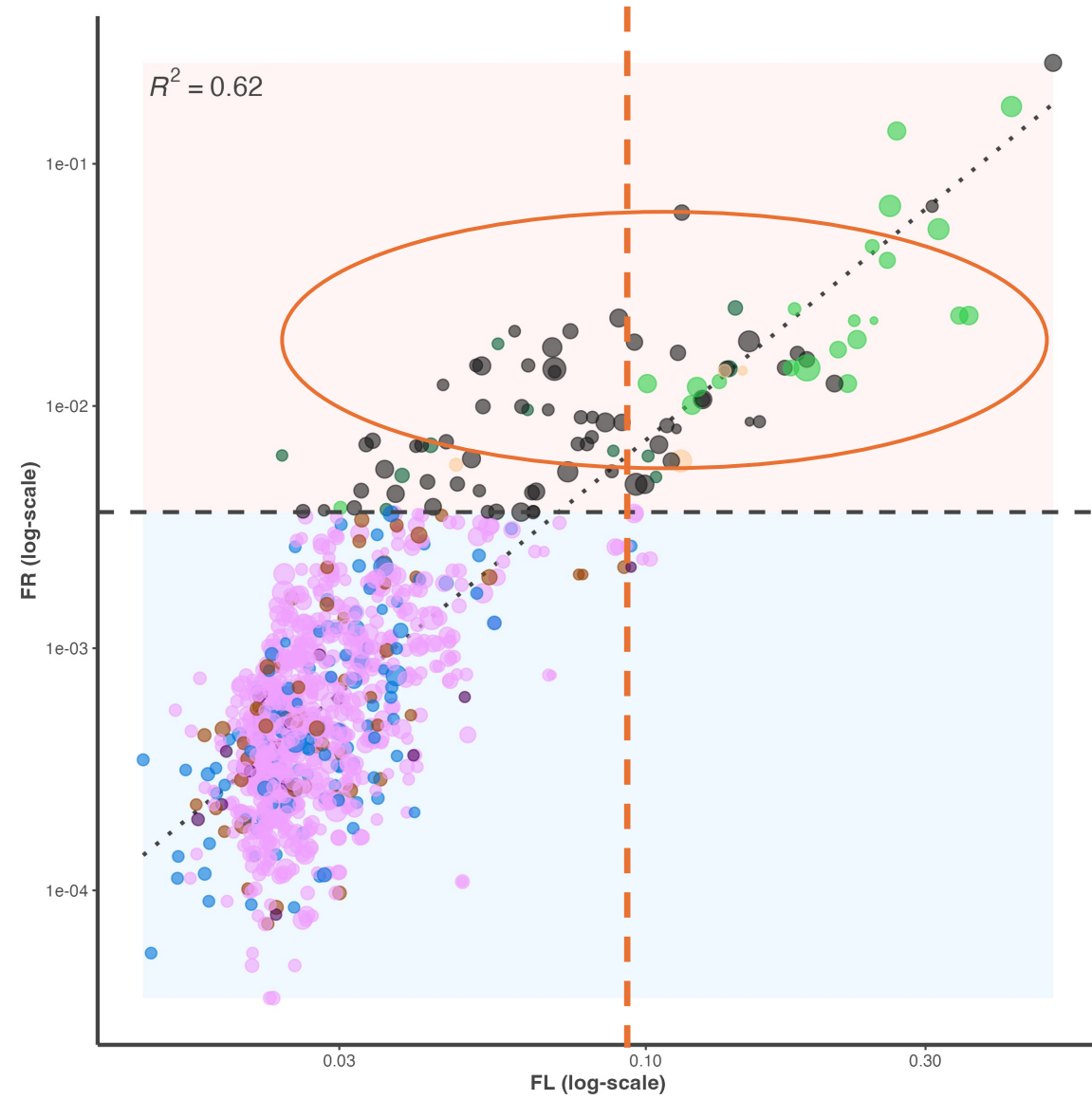
VS



$G_{RL} F_{RL} P_R$

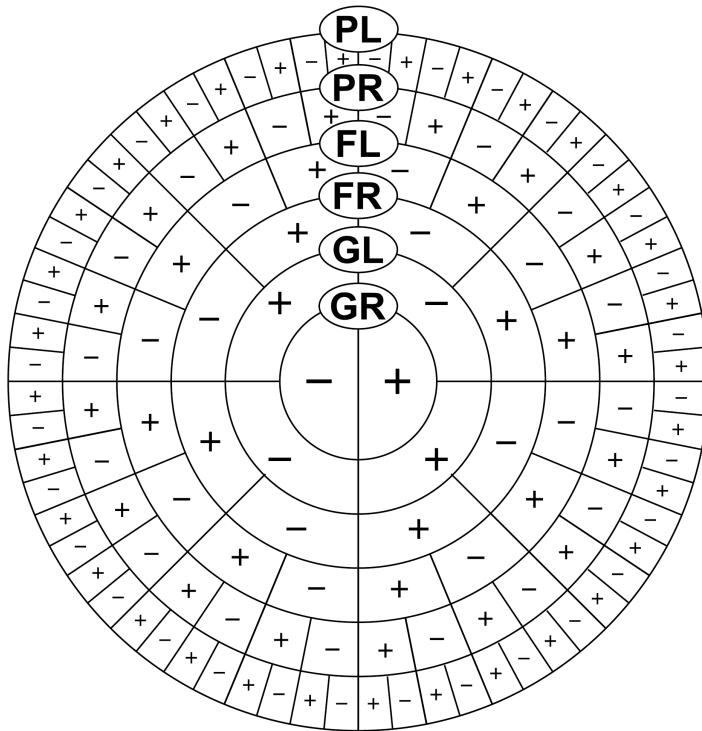




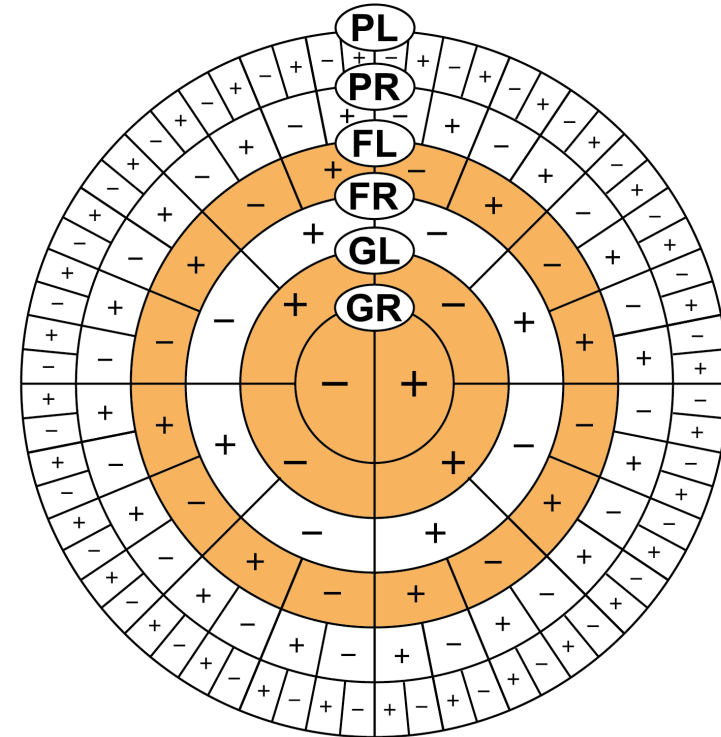
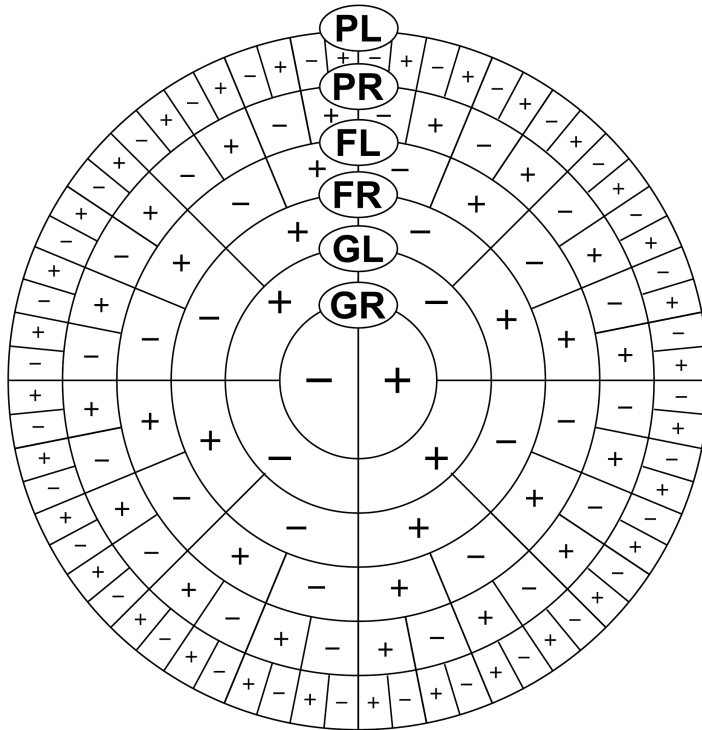


When and if rarity is best defined by its
comprehensiveness or by its utility in
explaining ecology?

When and if rarity is best defined by its
comprehensiveness or by its utility in
explaining ecology?



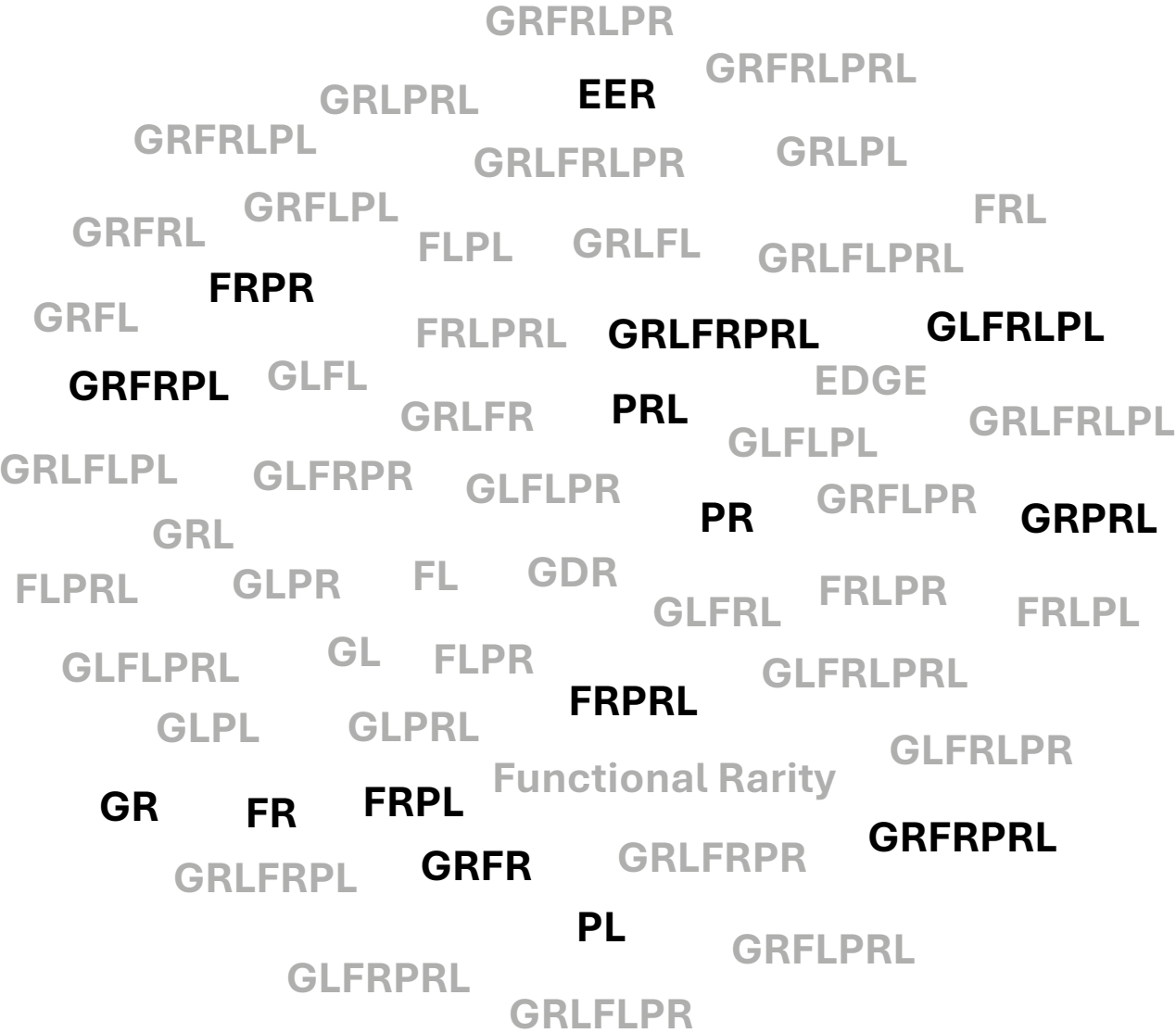
When and if rarity is best defined by its
comprehensiveness or by its utility in
explaining ecology?



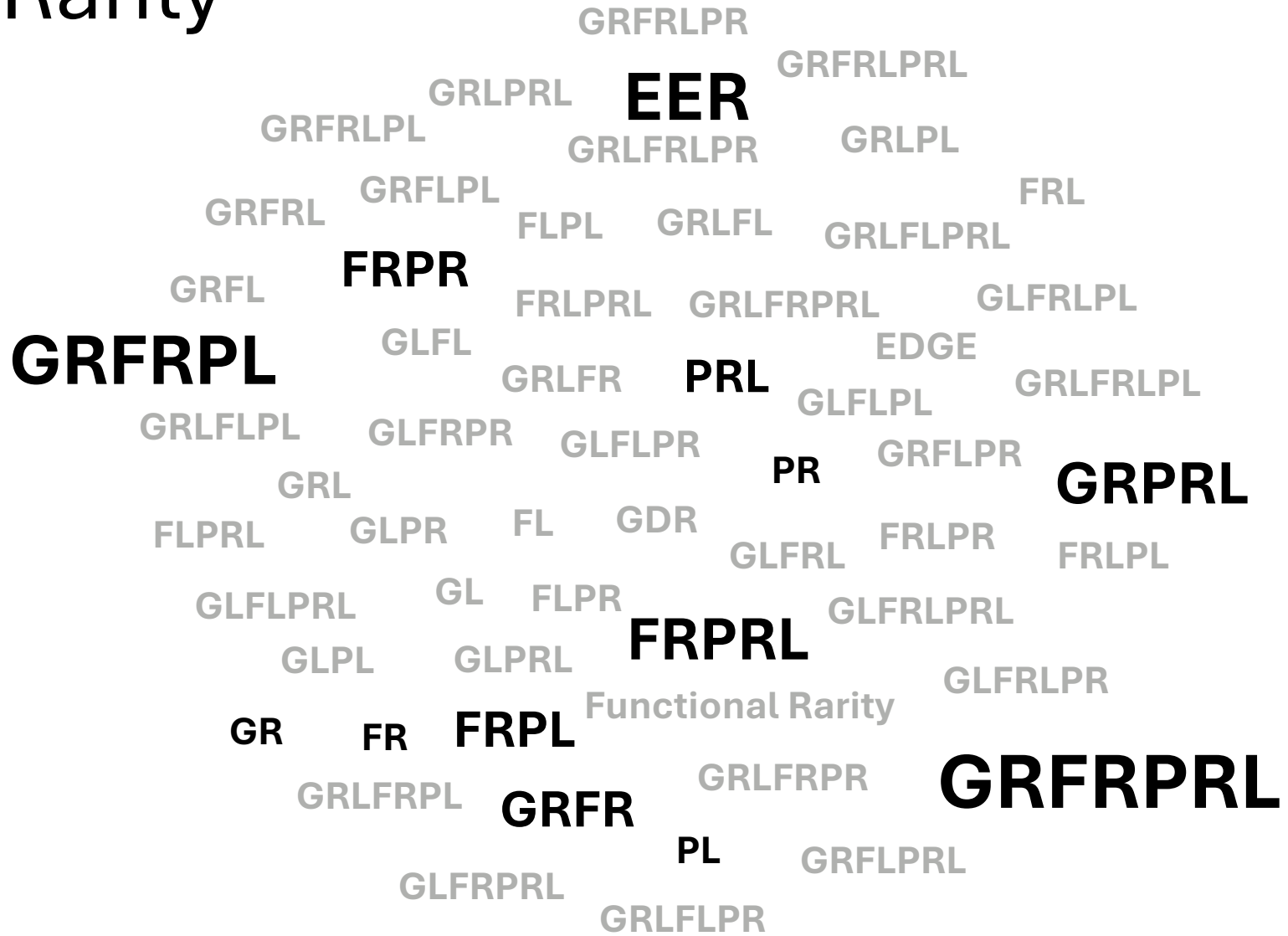
63 Restrictions

GRFRLPR
GRFRLPRL
EER
GRLPRL
GRFRLPL
GRLFLPRL
GRLPL
GRFRL
GRFLPL
FRL
FLPL
GRLFL
GRFLPRL
FRPR
GRFL
FRLPRL
GRLFRPRL
GLFRLPL
GRFRPL
GLFL
EDGE
GRLFR
PRL
GLFLPL
GRLFRLPL
GRLFLPL
GLFRPR
GLFLPR
PR
GRFLPR
GRPRL
GRL
FLPRL
GLPR
FL
GDR
GLFRL
FRLPR
FRLPL
GLFLPRL
GL
FLPR
GLFRLPRL
GLPL
GLPRL
FRPRL
GLFRLPR
GR
FR
FRPL
Functional Rarity
GRFRPRL
GRLFRPL
GRFR
GRLFRPR
PL
GRFLPRL
GLFRPRL
GRLFLPR

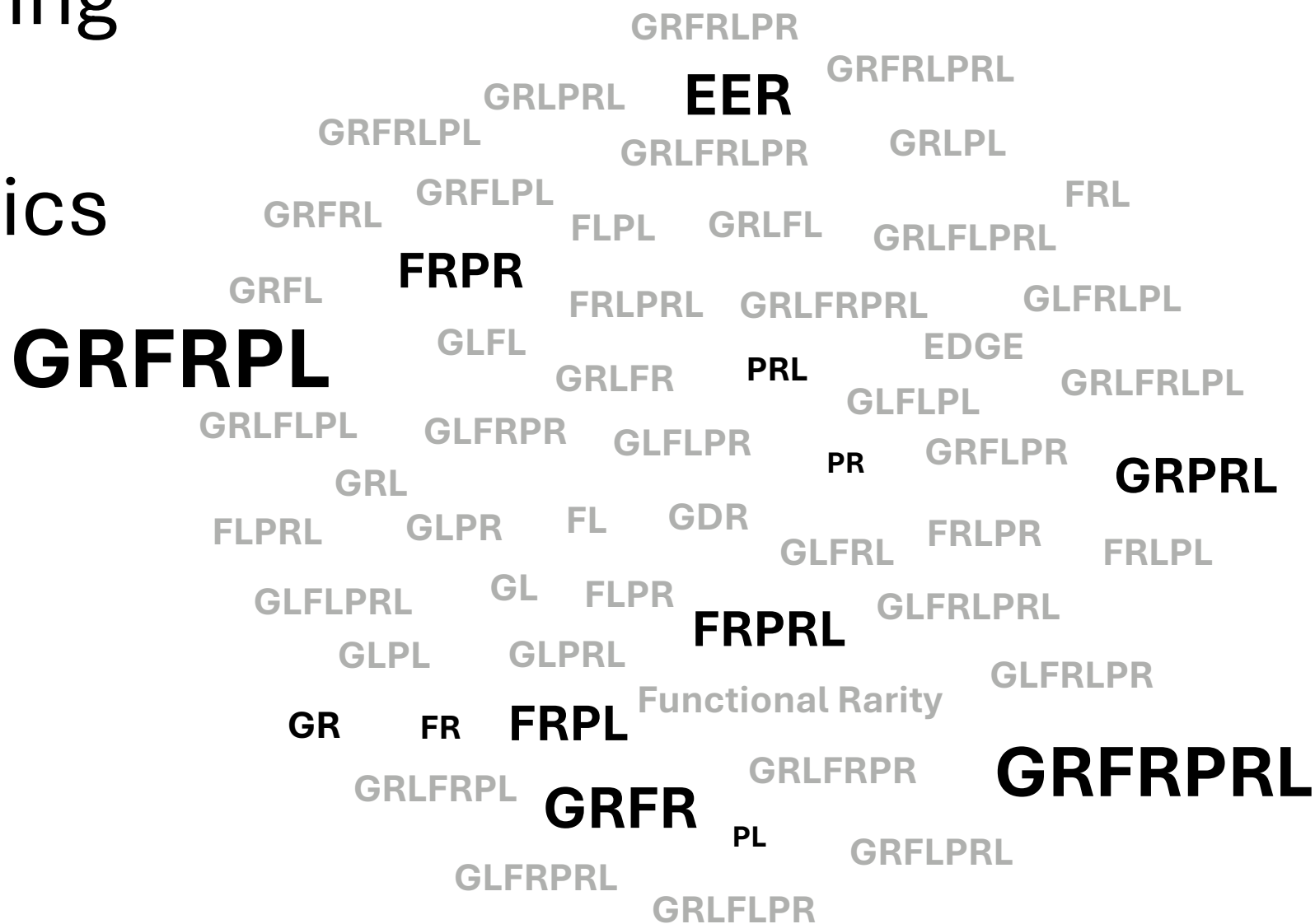
Limiting Abundance Data



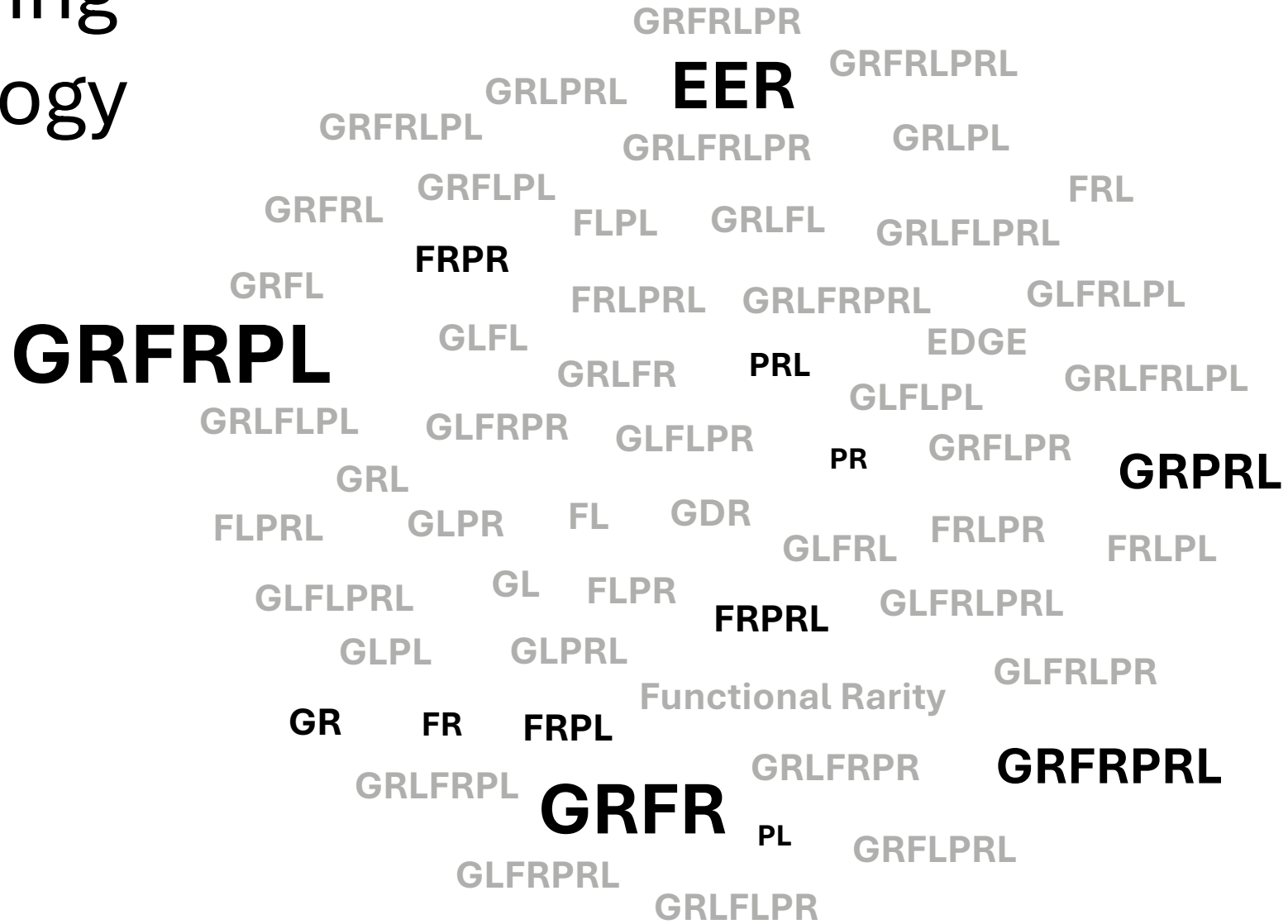
Defining Rarity



Explaining Range Dynamics



Explaining Phenology



1

Can we use rarity as a
tool to understand
biological processes?
How?

**Yes... with some
caveats**

2

Are rarity types robust
and ecologically
relevant?

1

Can we use rarity as a
tool to understand
biological processes?
How?

**Yes... with some
caveats**

2

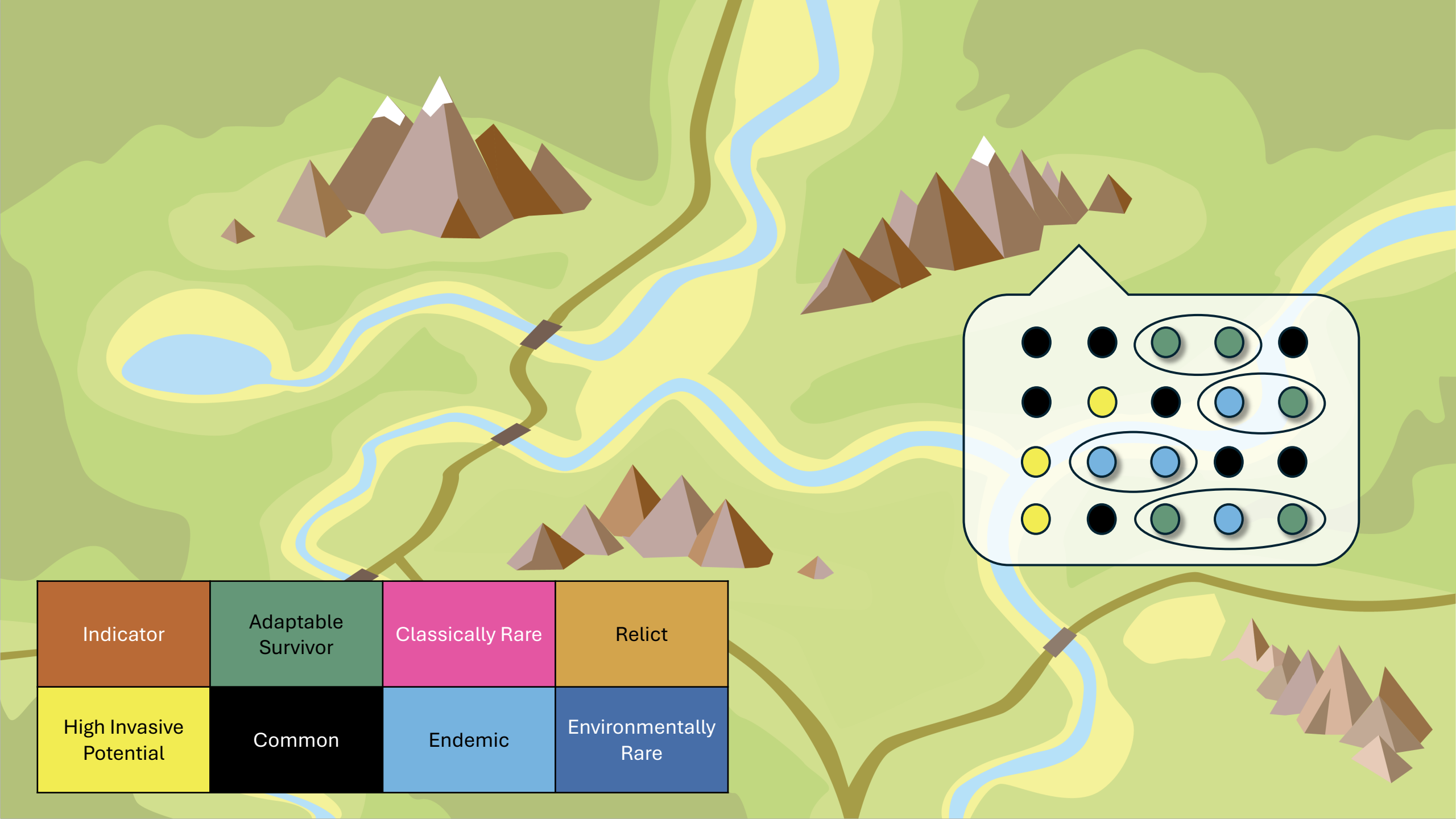
Are rarity types robust
and ecologically
relevant?

**Probably... with large
impacts for
conservation**

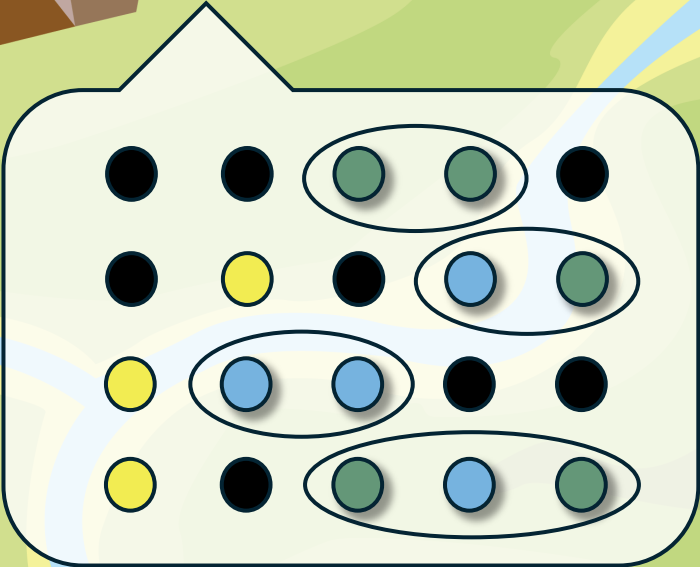


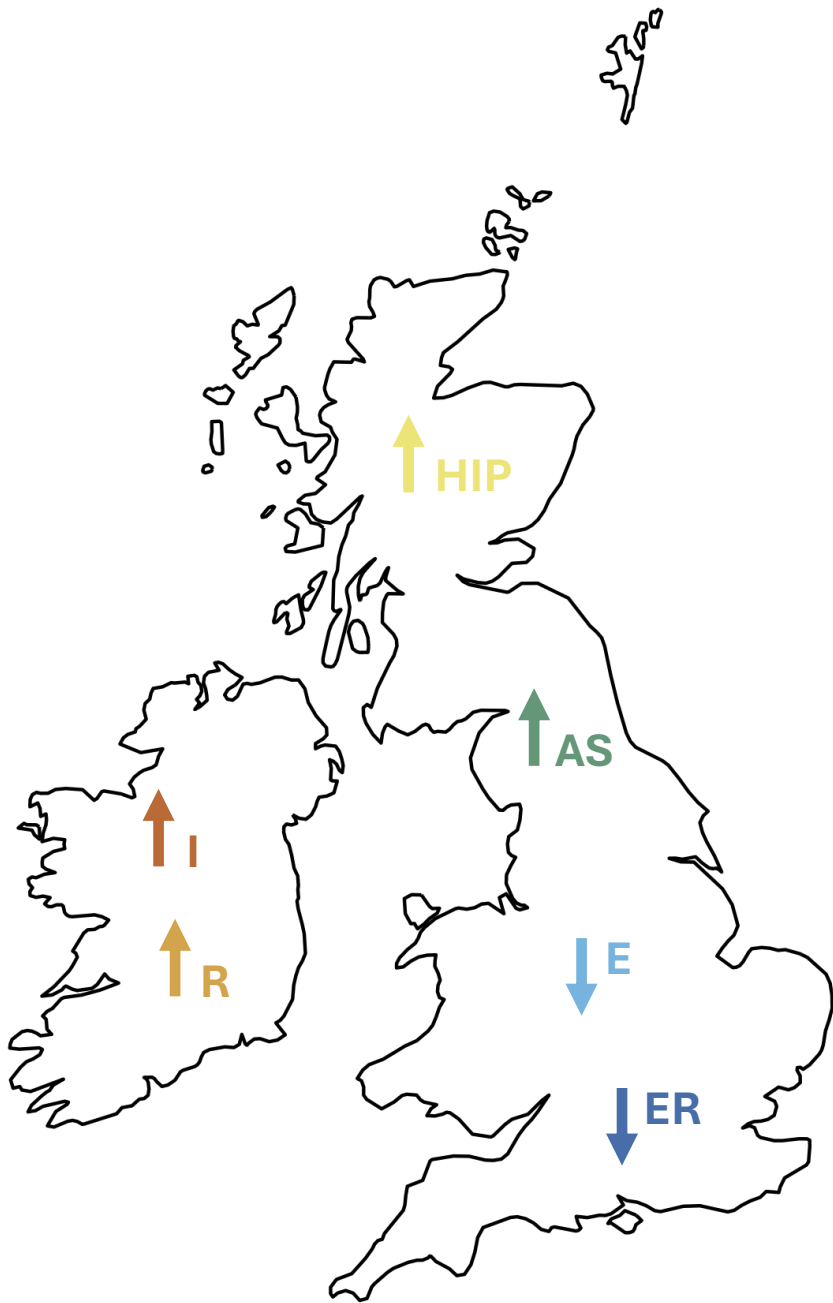
Orange	Blue	Dark Blue	Dark Blue	Orange
Orange	Blue	Orange	Pink	Dark Blue
Yellow	Pink	Pink	Orange	Orange
Yellow	Orange	Dark Blue	Pink	Dark Blue



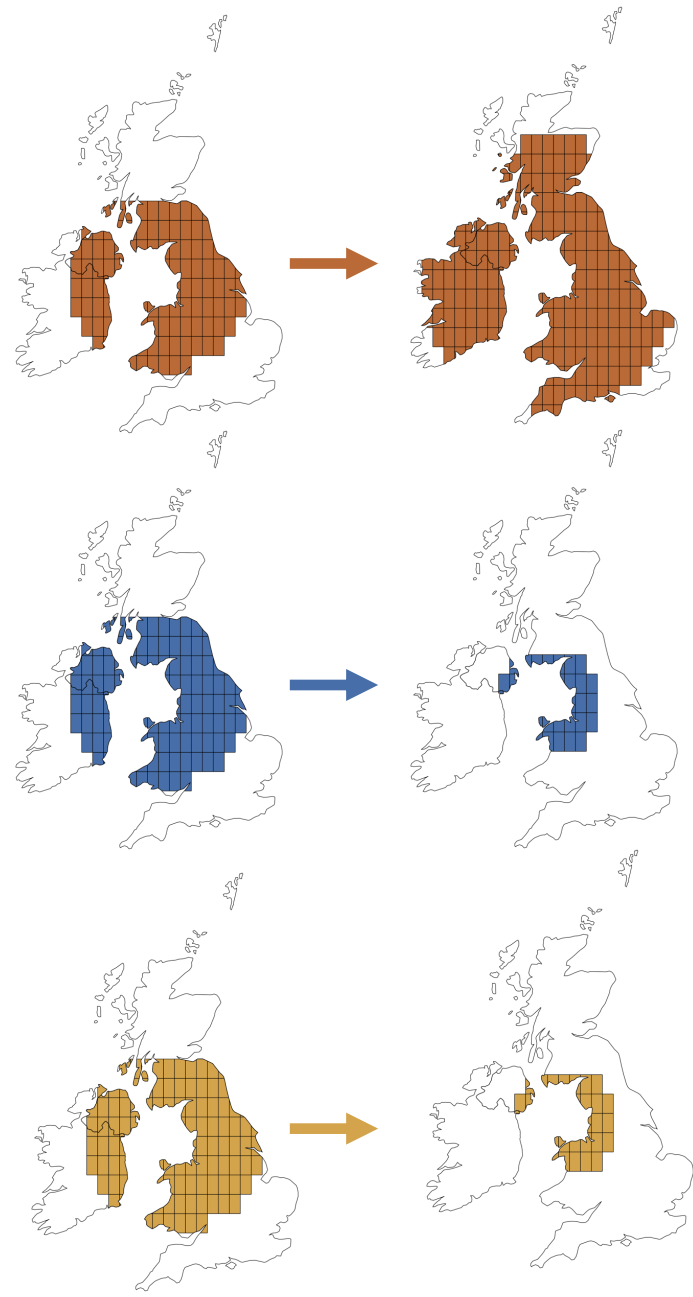


Indicator	Adaptable Survivor	Classically Rare	Relict
High Invasive Potential	Common	Endemic	Environmentally Rare





ENDEMIC
RELICT *In-situ
ENVIRONMENTALLY
RARE *Ex-situ



The **Global Model of Discretized Rarity** and its complementary restrictions overcome data limitations and provide more than 50 novel definitions of rarity and tools to understand ecological processes, grounded in a unified conceptual framework.

Future Directions



- Identify a suite of complementary restrictions that capture a wide range of ecological processes.
- Uncover the abiotic and biotic gradients that have shaped diversity in rarity within these restrictions through evolutionary time.
- Link rarity to ecosystem function.

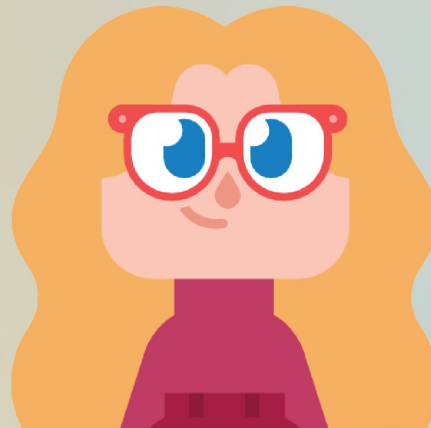
Predict how species, communities, and ecosystems respond to climate change using **biodiversity-rarity-ecosystem function (BREF)** relationships.

Thank you!

Beaulieu
Lab

O'Meara
Lab

Bailey
Lab



anytko@vols.utk.edu